Foundations for the Measurement of Phenomenal Symmetry* **

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With 18 figures and 1 table

1. Introduction

In many perceptual tasks there is a striking difference between human performance and the performance of machines even if they are designed for the task. Examples of such situations are: orientation in space, identification of objects, avoiding objects on a collision course, or decisions about where to grasp objects in order to handle them. What is common about these situations is that they are characterized by structural rules which are usually summarized under the heading of 'symmetry'. However, the common dictionary definition captures little of this general importance of symmetry:

> "Due or just proportion; harmony of parts with each other and the whole; fitting, regular, or balanced arrangement and relation of parts or elements; the condition of being well proportioned or well balanced. In stricter use: exact correspondence of size and position of opposite parts; equable distribution of parts about a dividing line or center". (Oxford English Dictionary 1971, p. 32o7)

This definition apparently falls into two parts: an evaluative or aesthetic definition (first section) and a constructive definition (second section). Both fail to make plausible why symmetry plays such an important role in everyday perceptual tasks which are either of high survival value or of high practical value. One further problem with this definition is that it implicitly assumes an easy classification of objects into either symmetric or unsymmetric forms. Real-world objects, however, are characterized by greater or lesser degrees of various kinds of symmentry but practically never by perfect symmetry. The goal of this study is to investigate how these degrees of symmetry can be measured such that the behavioral effects depending on them correlate with the derived measures. Two preconditions have to be met for this goal to be accomplished: First, the different forms of symmetry have to be unambiguously classified. And second, the level has to be identified on which symmetry is analyzed in the perceptual process.

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[*] The ideas underlying the research reported here have been developed while the author was visiting the Department of Psychology, Stanford University. From the fruitful discussions I have had in Roger SHEPARD's seminar when reporting these ideas I have very much gained insight in related topics. The theoretical framework owes very much to the long and thorough talks I have had with Jennifer FREYD. Gisela REDEKER has read an earlier draft and has suggested many helpful changes.

[**] Diesen Artikel möchte ich Herrn Prof. Dr. Herbert KALLINA widmen, der mir schon als jungem Studenten Sinn und Funktion von Formalisierungen in der Allgemeinen Psychologie nahegebracht hat.

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WOLF (1949) has shown that by combining the isometric operations of translation, rotation, and reflection and the homoeomorphic operation of stretching the symmetry patterns found in nature, as well as in decorative art, can be described in terms of algebraic group theory. Forms of symmetry relying on only one of the above mentioned operations are regarded as the basic examples of the different kinds of symmetry: repetition (due to spatial or temporal translation), bilateral symmetry (due to reflection or rotation in the third dimension), rotational symmetry, and central symmetry (due to stretching in all directions at the same time and with the same speed). Figures 1 a, b, c, and d show examples of these basic types of symmetry. These forms of

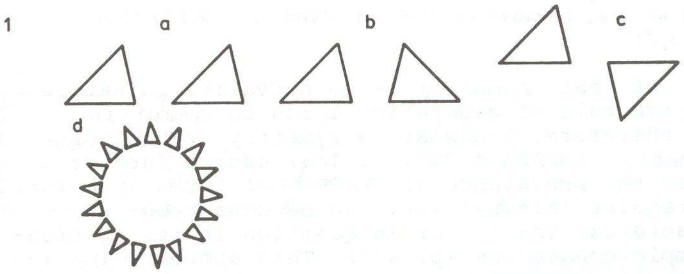

Fig. 1: Examples for (a) repetitive, (b) bilateral, (c) rotational, and (d) central symmetry.

Symmetry can be found to influence humand behavior differently in many situations: For instance, when either the observer or the observed object or both move in space, the detection of bilateral and central symmetry is crucial for processing information about the spatial orientation and the detection of paths for possible collisions (BALL and TRONICK 1971). Another example: For monitoring dynamic processes in space or time, picking up rhythmic information is necessary. Rhythm, however, can be regarded as a special form of translatory symmetry. And finally, technical processes depending on wheels or hinges exhibit rotational symmetry and can only be controlled effectively if the type of symmetry is identified correctly.

If the problem of veridicality in perception is approached from point of view of Critical Realism (METZGER 1954; SHEPARD 1981) there arises a double question concerning symmetry:

(i) How do symmetric forms emerge in nature?

(ii) How have the perceptual processes developed which account for the fast pick-up of symmetries in our environment?

The point of view of Critical Realism in perception is regarded here as opposed to (naive) Realism (see MACH 1903) and direct Realism (SHAW and TURVEY 1981; WILCOX and EDWARDS 1982) on the one hand and to Rationalism (GREGORY 1971; GOODMAN 1978, WHEELER 1979) on the other. In the first approach the position is taken that symmetry is characteristic for our environment and that our senses are tuned for the veridical perception of this environment. That is, the major problem for a scientific analysis of perception is answering question (i). From the Rationalist point of view human perception is regarded as imposing structure, namely symmetry, on what is physically given. The analysis of perception in this framework can therefore be described as the investigation of the structural and procedural characteristics of "the making of the world in our minds" (GOODMANN 1978). In contrast to these two positions SHEPARD (1981) in his principle of 'psychophysical complementarity' indicates how structural characteristics of the world and active perceptual processes interact:

> "(1) The world appears the way it does, because we are the way we are, and (2) We are the way we are, because we have envolved in a world that is the way it is" (p. 332).

WEYL (1954) points out that symmetry is so pervasive in nature because any repetitive rule of generation leads to symmetric patterns. For him, therefore, translatory symmetry is the most basic form of symmetry. KOFFKA (1935, p. 1o8) adds a further, physical, reason for the prevalence of symmetric forms by pointing out that they require 'minimal work' to be conserved: that is, they characterize the object in question in its stationary state under simple conditions (p. 4o9). This approach implicitly assumes bilateral symmetry to be the most basic type. LUDWIG (1949) argues that for the reasons of repetition and balance the detection of the type of symmetry characteristic for a pattern or contour can be assumed to be of high survival value for all animals including humans. Furthermore, repetition in the evolution of biological systems seems to be a very effective selection mechanism because it is stable against random perturbations.

The importance of symmetry in form perception has been pointed out especially by Gestalt psychologists. They coined the term 'Prägnanz' to describe the effects on perception which they assumed to be due to tendency of physical objects toward stationary states (KÖHLER 192o). KÖHLER and KOFFKA were strongly influenced by CURIE's work on symmetry (1884/1894) where he states:

> "Actions produced tend to be more symmetric than the causes" (p. 11o).

The stationary states are characterized by balanced distributions of forces, the effect of which can be described and is usually perceived, as symmetry or near symmetry (cf. METZGER 1954):

> "The elements given are grouped together so that the most symmetric, balanced and concentric wholes emerge" (p. 1o8-1o9, my translation).

Besides the fundamental philosophical question concerning the epistemological status of symmetry, the various examples for the function of symmetry in the regulation of adaptive behavior indicate that the detection of symmetry is of high practical value too. In contrast to the emphasis on symmetry in psychology, approaches to computer vision have not taken into account symmetries of forms until recently (BLUM and NAGEL 1978). The reason for this can be seen in the fact that the determination lines, centers, or planes cannot be done without knowledge about the type and orientation of symmetry exhibited by the form under analysis. The human observer has apparently no difficulty in combining the bottom-up and top-down search necessary for jointly picking up the parts of a visual scene and interpreting the underlying type of symmetry, whereas an algorithmic description of this task is very complicated. BLUM and NAGEL attack this problem by confining their analysis to bilateral symmetry and by using the symmetry information entirely for the segmentation of the visual scene. Their algorithm (1) consists of determining the line on which the centers of all maximal circles lie which can be inscribed into the contour of a given form. Figure 2 demonstrates that this procedure not only fails to detect the axes of symmetry

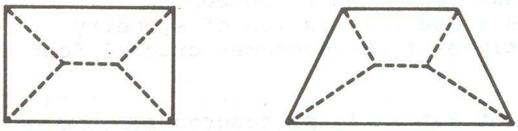

Fig. 2: Description of forms (rectangle and trapezoid) by the lines on which the centers of the maximal circles lie which can be inscribed into these forms.

but furthermore characterizes perceptually distinctive forms by highly similar characteristic lines. BRADY (in press) suggests a different approach which overcomes these problems, at least partially, by an algorithm relying on smoothed local symmetries. He too, however, confines this analysis to bilateral symmetry (see Figure 3). A further problem with this approach is that it does not provide

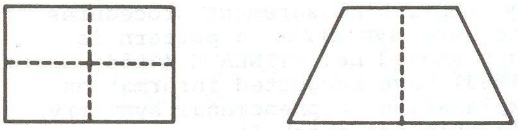

Fig. 3: Smoothed symmetry lines for the forms in Figure 2

a measurement procedure for the degree of symmetry in a given form.

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[1] BLUM (1973) has suggested this 'grass-fire' transform for biological shapes. PSOTKA (1978) reports experimental results indicating that people asked to put dots inside a given form, tend to put them in locations close to the lines due to a 'grass-fire' transformation of the form.

2. The Definition of Symmetry in its Relation to the Measurement of Symmetry

The problem of measuring the degree of symmetry in a given form is influenced by the definition of symmetry underlying the analysis: If one assumes that the most elementary form of symmetry is repetition (MACH 19o3; WEYL 1952), then the measurement procedure can consist of comparing the repetitive elements to the total number of elements. Such a procedure can work automatically. If, however, reflection is assumed to constitute the basis of phenomenal symmetry as it is done by Gestalt psychology, then the search of the reflection planes describing the symmetry of a given form becomes a precondition for determining the amount of violations and thereby the degree of symmetry. A procedure based on the latter conception of symmetry can only be automatic on a very low level of analysis (identification of basic form) but on higher levels it has to be led by perceptual hypotheses because on higher levels the orientation of symmetry planes and the structural relations of them becomes crucial for phenomenal symmetry.

In the following part a couple of automatic procedures for the measurement of symmetry are reviewed. Their failure to capture the characteristics or phenomenal symmetry leads to the assumptions of a multiple-level process of human symmetry perception. In the framework of group-theoretic analyses of symmetry is defined as a sequence of reflection operations and measurement procedures are proposed which are in line with this definition.

2.1. Approaches for the Automatic Measurement of the Degree of Symmetry

The definition given in the Oxford English Dictionary (see, for instance, terms like "exact correspondence" or "equable distribution") suggests that symmetry can be thought of as a special case of redundancy according to information theory. This interpretation of symmetry implies measurement procedures in a straightforward manner: The more symmetric a pattern is, the lower the conditional entropy should be. ATTNEAVE (1954, 1955) and CARNER and CLEMENT (1963) have suggested information theoretic approaches for the explanation of phenomenal symmetry. In order to measure symmetry in this framework it is necessary to define a unit of analysis in advance (e.g. by superimposing a grid) and to determine the level of analysis. The level of analysis is determined by the order of the conditional probabilities which are to be taken into account. One could argue that human vision too depends on predetermined units of analysis as given by the biological constraints of the sensory system and something analogous to a hard-wired level of analysis (e.g. lateral inhibition and receptive fields). However, for these information theoretic approaches to be computationally feasible, the grid has to be so coarse that simple changes in the orientation of the grids can have vast effects on the characteristics of the form in question (cf. Figure 4 (a) vs. (b)).

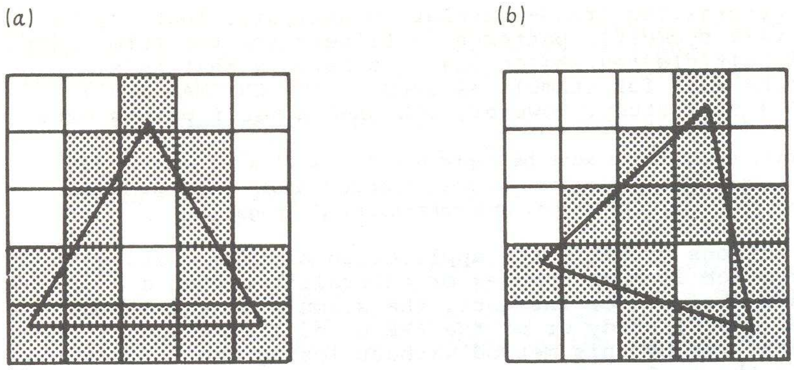

Fig. 4: An equilateral triangle seen through a 5 *by* 5 -grid. (a) in standard orientation (b) after a rotation of 2o°.

However, the determination of the proper level of analysis in larger grid which could be assumed to model characteristics of receptors, has to be done by the experimenter in order to avoid computational problems due to the combinatorial character of the problem.

2.1.1. *Autocorrelation Theory of Form Detection*

UTTAL (1975) and others have tried to overcome the latter problem in the following way: In a given grid the grey values of the cells are measured, then for each column and row of the grid autocorrelation functions for these values are computed. It is assumed by UTTAL (1975) that the characteristics of the two-dimensional autocorrelation plot allows for an automatic detection of the symmetries characteristic for the form under analysis. However, FOX and MAYHEW (1979) have demonstrated that the Autocorrelation theory of form detection fails to recognize patterns which have been slightly but systematically changed, for instance, by tilting or non-linear stretching.

2.1.2. *An Entropy Approach to the Measurement of Symmetry (Symmentropy)*

YODOGAWA (1982) starts from the information theoretic approach as proposed by ATTNEAVE (1954, 1955) and HOCHBERG and McALISTER (1953). He combines it with GARNER's (1962) pattern analysis and proposes a general measurement procedure for patterns which can be represented in a $2^{\rho} \times 2^{\rho}$ -grid. The measurement procedures consists of Walsh functions, which can be regarded as equivalent to Fourier (sine-cosine) functions. For each grid size the Walsh functions for all purely symmetric forms in that grid are generated. YODOGAWA discriminates between the following forms of symmetry: double symmetric (bilaterally symmetric in both dimensions), horizontally symmetric, vertically symmetric, and centrosymmetric (rotationally symmetric with an angle of 18o degrees). The Walsh functions of the actual forms are then compared to the Walsh functions for purely symmetric forms by means of a generalized cross-correlation analysis. That is, he uses the purely symmetric patterns as filters for the form under analysis. A multiple regression analysis reveals that this approach works well for stimuli as used in the CHIPMAN (1977) or the HOWE (1980) study. However, YODOGAWA himself points out:

> "(1) In general, any pattern must be represented by a 2^ρ x 2^ρ-matrix (2) The two-dimensional Walsh transform is not invariant under each operation of translation, rotation, dilation, and contraction." (page 236)

These restrictions prohibit the application of this measure to any form which is more complex or naturalistic than a checker-board design. For instance, the stimuli of the PALMER and HEMENWAY (1978) study or of the FREYD and TVERSKY study cannot be analyzed by this method without losing the information contained in the angles, bars, or line segments which are the natural constituents of their stimuli. Figure 2 (a) demonstrates the amount of distortions figurative elements undergo if they are subjected to the transformation into grid-patterns. The impact of even slight rotations of the orientation of the form for the resulting grid pattern can be seen in Figure 2 (b).

The failure of the symmentropy approach for complex forms can be attributed to its critical restriction to patterns which can be represented in grids of a given orientation. A way to overcome this restriction would be to represent forms by patterns of point and to determine the distances between them. The actual analysis would then be done on the basis of the pattern of distances instead of the actual positions of the points, because the pattern of distances is invariant under rotation. Moreover, if a standardizing procedure is performed, the size dependency of the symmentropy approach could also be avoided in an analysis of patterns of distances.

2.1.3. Proximity Analysis of Textures Exhibiting Symmetry

FOX and MAYHEW (1979) have developed what they call a "proximity analysis" for the detection of homogeneity vs. inhomogeneity in visual textures. The analysis is done in the following steps:

(i) picking out the points which make up the texture,

(ii) measuring the proximity of each point to all the points in its neighborhood (the p-structures of all points),

(iii) comparing the p-structures for any two points, and counting the number of matches.

The number of matches gives the ρ-measure for the texture under analysis. FOX and MAYHEW have shown that when starting from a broad neighborhood and then proceeding with ever narrower neighborhood the grade of separation between the rho-functions for homogeneous and inhomogeneous textures correlates with the empirical detection time data.

While proximity analysis works well for texture analysis its suggested application to the perception of symmetric and nearly symmetric forms is problematic for the following reasons:

(i) FOX and MAYHEW (1979) do not define unambiguously what a 'point' is (p. 84);

(ii) the proximity operator is not restricted by any metric constraints;

(iii) they do not allow for random fluctuations, because the ρ-measure counts only the perfect matches. This feature makes it difficult for this theory to account for the results reported by the authors themselves;

(iv) they do not give any straightforward generalization for this measure beyond the area of texture perception.

The discrimination between translatory and bilateral symmetry is done by comparing ρ as a functions of the diameter of the neighborhoods for the different types of symmetry. Since the forms of these functions are influenced by the texture, no general criterion for the discrimination between types of symmetry can be provided by the ρ - metric. (2)

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[2] Without a careful choice of the size of the neighborhoods and without a fitting definition of 'points' the difference between translatory and bilateral symmetry does not become apparent. The dependency on the starting position combined with the ambiguity in the definition of 'points' prevents the ρ - measure from being an automatic detector for the type and amont of symmetry in forms.

A generalization of FOX and MAYHEW's measure should provide rules for the definition of 'points', a restriction of possible proximity operators (e.g. Euclidean distances), a threshold model for the determination of matches, and at least rules of thumb for the determination of the diameter of the neighborhood to start with. After these preliminary definitions it would be possible to define a metric which captures the differences between the ρ -functions characterizing different textures.

One problem with the proposed generalization of FOX and MAYHEW's ρ -measure is that it would result in even more comparisons of permutations of points than in their original analysis. FOX and MAYHEW report that already the original ρ - measure necessitates more comparisons for their textures than could be done on their computer. Furthermore such a generalization would be of questionable value for the perception and analysis of contours which are not primarily determined by textural features.

My general criticism to the automatic approaches symmetry measurement can be summarized in the following way: In these approaches either the 'granularity' of a physical representation of a given form is arbitrarily predetermined or a fixed grid with a given orientation is superimposed on the forms. Lines, regular curves, etc. are not analyzed as such. What is analyzed instead, are the intensity or point patterns they give rise to. These patterns can be regarded as consisting of coordinates in fixed units as, for instance, 'pixels' in computer vision. However, the apparent flexibility by which humans detect symmetry in forms cannot be captured by these measures because of the described restrictions. Furthermore, the ability of human perception to structure forms according to different types of symmetry relies on a previous segmentation of the visual scene into elementary shapes like bars, lines, or smooth curves.

3. A Dual Process of Symmetry Perception

In order to account for the characteristics of phenomenal symmetry as well as for the apparent success of BRADY's (in press) approach to low-level pattern recognition by means of smoothed local (bilateral) symmetries' I assume that the processing of symmetry in human perception takes place on at least two levels:

(i) Elementary forms are identified by means of automatic analyses of uninterpreted input data. The findins on receptive fields (HUBEL and WIESEL 1962) and on the detection of symmetry without form detection (LOCHER and NODINE 1973) indicate that such automatic symmetry detectors exist in the visual system. If these detectors can be modelled by BRADY's (1982) or YODOGAWA's (1982) approaches they have to be sensitive to orientation and size.

(ii) By analyses on a higher level, the elementary forms identified on level (i) are then interpreted as parts of complex visual scenes. These scenes can be assumed to be structured by the symmetries exhibited

Dual-process models for the perception of symmetry have been proposed by JULESZ (1971), BRUCE and MORGAN (1975), and PALMER and HEMENWAY (1978), PALMER (in press). For instance, JULESZ (1971) compares rapid, holistic, and immediate processes ('perception' in his terminology) to slow, component-wise, and cognitively mediated processes ('scrutiny'). BRUCE and MORGAN (1975) report empirical evidence for an automatic processing of vertical bilateral symmetry whereas they claim that non-vertical bilateral symmetry, repetition, and rotational symmetry are detected by higher processes. These results imply that bars, lines, circles, and similar shapes are taken as basic constituents of the forms under analysis.

The phenomenal symmetry can be analyzed as the interaction of three subprocesses:

(i) detection of the axis or the center of symmetry,

(ii) detection of the type of symmetry (translatory, rotational, bilateral, central),

(iii) detection of the amount of symmetry.

It is further assumed that this phenomenal symmetry is decisive for the alleged effects of symmetry on the memory of forms, the spatial orientation of the perceiver during movement or in relation to objects, and the aesthetic value of symmetric forms. The measures of symmetry as proposed by UTTAL, FOX and MAYHEW, YODOGAWA have in common that they do not differentiate between these two levels in symmetry perception (in contrast to BRADY, in press). The proposed differentiation of the perception of symmetry into two levels of processing is in line with MARCEL's (in press) distinction between conscious and unconscious levels of perception.

3.1. Measuring the Degree of Symmetry on the Basis of the Underlying Group Structure

In order to develop measures for the degree of symmetry in this framework, it is necessary to describe the structural and formal characteristics of the different types of symmetry. SPEISER (1925), ENGELHARDT (1949), and WOLF (1949) have investigated the possibility of describing symmetries by the underlying group-theoretic structures. WEYL (1952) has integrated these approaches and has applied them to the analysis of decorative art, mineralogy, and biology.

3.1.1. Group Theoretic Description of Types of Symmetry

ENGELHARDT (1949) motivates the analysis of symmetry on the basis of algebraic group structures by pointing out:

> "Most of the observations in physics done in order to arrive at general laws aim at the detection of symmetries. Often the symmetry is quite remote and one has to be cunning to discover it. Symmetry is not immediately given by experience, resembling more a hidden blueprint. It underlies the real structures and shines somewhat through the apparent disorder." (p. 17, my translation).

He defines the following symmetry operators: translation (T), horizontally bilateral symmetry (M_h), vertically bilateral symmetry (M_v), rotation (R), and translation coupled with horizontally bilateral symmetry (G). He shows that T and R form algebraic groups.

Operator T

(i) Any combination of translation is again a translation.

(ii) The translation by zero is the neutral element.

(iii) The reversion of the direction of translations is the inverse.

Operator R:

(i) Any combination of rotations is again a rotation.

(ii) The rotation by 0°, by 360°, or by multiples of 360° gives the neutral element.

(iii) The inverse of any rotation by α is given by the rotation by $360^{\circ}-\alpha$.

ENGELHARDT (1949) concludes that bilateral symmetry does not exhibit the structure of an algebraic group and he implies that for the reason bilateral symmetry has to be treated separately and does not lead to the detection of underlying structures as the other forms of symmetry. This conclusion is only correct if the symmetry operations are restricted to operations in the plane. However, research in visual perception (METZGER 1936/1975; GREGORY 1971; SHEPARD 1981) indicates that human vision usually imputes spatial, that is, three-dimensional patterns even if they are not given. In many cases optical illusions can be traced back to an underlying spatial interpretation of two-dimensional arrays. Given this tendency of perception towards three-dimensional interpretations, bilateral symmetry can be treated as rotation through space. In this case the same group-theoretic interpretation as for planar rotation can be applied to bilateral symmetry.(3)

WOLF (1949) classifies the symmetry operators in a slightly different way. He discriminates between isomorphic operators (translation, rotation, reflection, plus any combination of them) and homoemorphic operators (expansion plus any combination of expansion and isomorphic operators). The mapping functions for the operators can be derived in a straightforward manner and can later on be used to develop indices for the amount of symmetry due to the different symmetry operators:

Translation:

$$_2x_i = {}_1x_i + a \tag{1}$$

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[3] SHUBNIKOV and KOPTSIK (1972, 1974) provide a general group-theoretic framework for symmetry, antisymmetry, and generalized symmetry. The applications of these analyses to decorative art, crystallography, biology, and engeneering imply, however, that symmetry has already been detected. They discuss 'relative equality' as a basis of symmetry but take for granted that exact matching of parts can be achieved.

$$ {}_2y_i = {}_1y_i + b \qquad (2) $$

Rotation by $\phi = 180^\circ$ around the center of a coordinate system:

$$ {}_2x_i = {}_1x_i \times \cos\phi + {}_1y_i \times \sin\phi; \qquad (3) $$

$$ {}_2y_i = {}_1y_i \times \cos\phi + {}_1x_i \times \sin\phi. \qquad (4) $$

Reflection along a line $y = x \times \tan\left(\frac{\phi}{2}\right)$:

$$ {}_2x_i = {}_1x_i \times \cos\phi - {}_1y_i \times \sin\phi; \qquad (5) $$

$$ {}_2y_i = {}_1x_i \times \sin\phi - {}_1y_i \times \cos\phi. \qquad (6) $$

Expansion:

$$ {}_2x_i = k \times {}_1x_i; \qquad (7) $$

$$ {}_2y_i = h \times {}_1y_i. \qquad (8) $$

'k' and 'h' may be functions of the index i. Linearly or exponentially increasing or decreasing functions combined with sinusoidal functions seem to be sufficient for the description of phenomenal symmetry which gives rise to or maps dynamics. If one defines expanding symmetry for pairs of orthogonal functions, for instance, sine and cosine, expanding symmetry can be described by an algebraic group structure, too.

WOLF does not analyze central symmetry. However, central symmetry can be looked upon as a special case of bilateral symmetry, because the form is symmetric for all directions in the plane. The mapping function for central symmetry can be generated by using polar coordinates instead of Cartesian coordinates. The symmetry underlying moving objects can be modelled by points moving away or to a focal point (the center) with equal speed: it is a combination of central symmetry and expanding symmetry.

As pointed out by ENGELHARDT (1949), the types of symmetry which give rise to group-theoretic descriptions of physical laws, are quite remote. Therefore it is not sufficient to measure the degree to which a given form is in agreement with a group-theoretically defined type of symmetry (the 'objective' degree of symmetry in a given form). It is also necessary to define salience functions for the types of symmetry and the orientation of the forms under investigation (the 'phenomenal' strikingness of symmetry in a given form). In the following part of this article indices will be proposed that measure the 'objective' degree of symmetry. By using these indices the 'phenomenal' strikingness of types of symmetry will be investigated empirically and suggestions for the determination of saliency functions will be made.

3.1.2. *Indices for the 'Objective' Degree of Symmetry*

The general idea underlying the proposed indices is to start from the line or the center of symmetry and to measure the amount of discrepancy between the coordinates as predicted by the mapping functions (see above) and the actual coordinates. Since the theoretical considerations about the processing of perceived symmetry and the reported empirical results (CORBALLIS and RORDAN 1975; PALMER & HEMENWAY 1978) indicate that bilateral symmetry can be regarded as the most natural type of symmetry, at first indices for this type of symmetry are developed.

For forms suggesting an axis of symmetry it seems to be a sensible way to measure the degree of symmetry by computing *the variance of elongation orthogonal to the axis not accounted for by symmetry*. This index models KOFFKA's notion of (bilateral) symmetry as the most stable state of a form. It is related to ATTNEAVE's (1955) and GARNER's (1975) information-theoretic measures for symmetry or goodness-of-form, because variance not accounted for by symmetry is a monotonic function of entropy. However, for the reason given, an index is chosen here which is tailored to the case of bilaterally symmetric forms, consisting of distinct sub-patterns (see Figure 5). The degree of symmetry in this form is captured by:

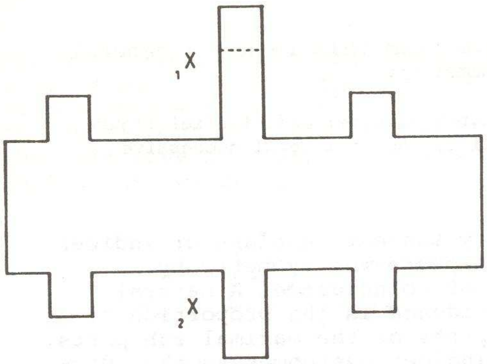

Fig. 5: An example for a nearly bilaterally symmetric form consisting of distinct subpatterns. This form is a modified version of stimuli used in the FREYD & TVERSKY study. The symbols refer to the index S_l.

$$S_l = \frac{\sum_{i=1}^{i=n} \frac{({}_1x_i - {}_2x_i)^2}{MAX({}_1x_i^2;\ {}_2x_i^2)}}{n} \qquad (9)$$

From a statistical point of view this index has many advantages. However, if one assumes that the detection of local symmetry (KIMCHI and PALMER 1982; FREYD and TVERSKY) depends on a process by which one part of the form is mapped into another part of it, then the index S_l does not reflect this process.

VIOLA (1904) has proved that all isomorphic symmetry transformations (reflection, rotation, and translation) and the combinations of the (e.g. screw rotation and glide reflection) in finite forms can be reduced to successive reflections in maximally three planes (see Figure 6).

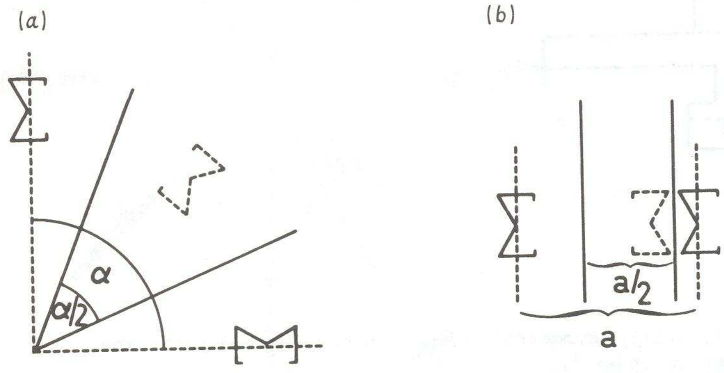

Fig. 6: (a) rotation by α^o as reduced to two consecutive reflections, which include an angle of α (b) translation by 'a' as reduced to two consecutive Σ reflections which are $\frac{a}{2}$ apart.

SHUBNIKOV and KOPTSIK (1974) derive from this result a general transformational definition of symmetry:

> "The label 'symmetrical' is applied to every (finite) or(infinite) figure which may be made to coincide with itself by one or several successive reflections in planes." (p. 127)

This general definition of symmetry suggests a class of indices for the degree of symmetry which capture the symmetry by counting the amount of violations of coincidence. A natural measure for the violation of coincidence is the proportion of the matching parts to the entire parts or the maximal sub parts. The application of this class of indices overcomes another disadvantages of the predominatly statistically motivated index S_1. This measure takes only into account violations of symmetry orthogonal to the axis of symmetry, but not violations due to shifts parallel to the axis of symmetry. The shifts, however, can be accounted for by additional reflections in the indices of symmetry which are derived from SHUBNIKOV and KOPTSIK's (1974) definition of symmetry.

In order to measure violations of bilateral symmetry in both directions it is necessary to define a search algorithm for shifted correponding parts of the form under examination and a metric of symmetry, which contains a penalty function for shift.

The proposed algorithm starts with a downward search from the upper end-point of the figure along the axis of symmetry. If a mismatch between two opposing parts of the form is found, a further downward search for a better match is started. For every better match the indexed part of S_M is computed (see below), which takes into account the amount of mismatch (proportion of opposing parts) and the amount of shift (proportion of parallel vs. orthogonal shift as given by the cosine of the direction of shift). Figure 7 gives an example of a nearly bilaterally

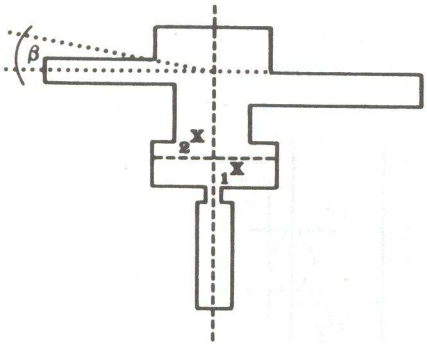

Fig. 7: A nearly bilaterally symmetric form with one shifted part. The symbols refer to index S_M.

symmetric form with a shifted sub part and illustrates the application of index. S_M. The pair of bilateral parts of the form with the highest value are preliminarily assumed to be the corresponding parts of the form. As long as no parts of the form have been assigned to more than one corresponding part S_M is computed.

$$S_M = \frac{\sum_{i=1}^{i=n} \frac{_1x_i}{_2x_i} \times \cos\alpha_i}{n} \tag{10}$$

In the case of multiple assignments, S_M is computed for any permutation of multiple assignments. The permutation gaining the highest value for S_M is subsequently taken as representing best the bilateral symmetry of the form under examination.

Bilateral symmetry can be regarded as a special case of central symmetry as having only one axis of symmetry instead of infinitely many axes intersecting at the same point. It is possible to capture this difference and the structural equivalence at the same time by using polar coordinates instead of Cartesian orthogonal coordinates. In this system every point, x_1, is unambiguously defined by its distance to the center, r_1, and its angle, θ_1. In this system of coordinates measuring the degree of symmetry in a form can be done analogous to index S_{11}. (see Figure 8)

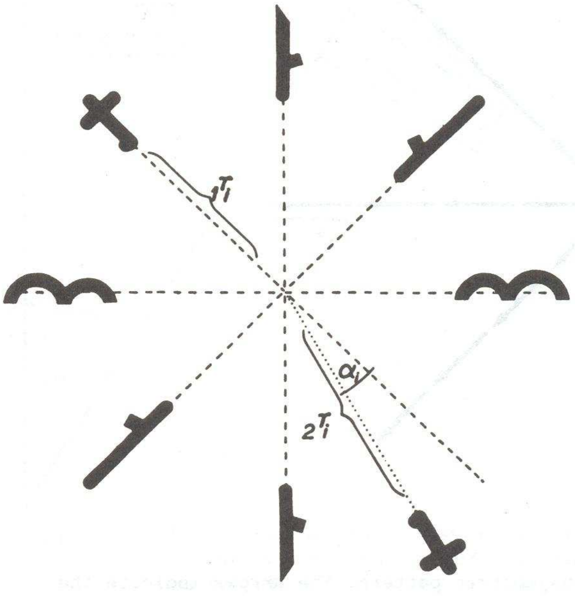

Fig. 8: Example of a form which is nearly centrosymmetric. The symbols refer to the index S_C.

The main difference between the procedures to determine the index consists in direction of the search procedure for equivalent subparts of the form: In the case of central symmetry the search follows the angle θ_1, whereas in the bilateral case the search follows a line.

$$S_C = \frac{\sum_{i=1}^{i=n} \frac{{}_1r_i}{{}_2r_i} \times \cos\alpha_i}{n} \qquad (11)$$

Special types of symmetry, for instance, the symmetry underlying the structure of snowflakes or Islamic ornaments, can be regarded as lying in between the bilateral and the centrosymmetric case.

Index S_C captures the central symmetry for statistic forms. However, as the results of the BALL and TRONICK study (1971) indicate, the case of dynamic central symmetry is of even higher survival value for the organism, for instance, for determining if an object is on a colliding path with the observer or not. Dynamic symmetry can be modelled by comparing the corresponding velocities, $v r_1 \ o_1$, instead of the positions of corresponding sub parts (see Figure 9). The index for

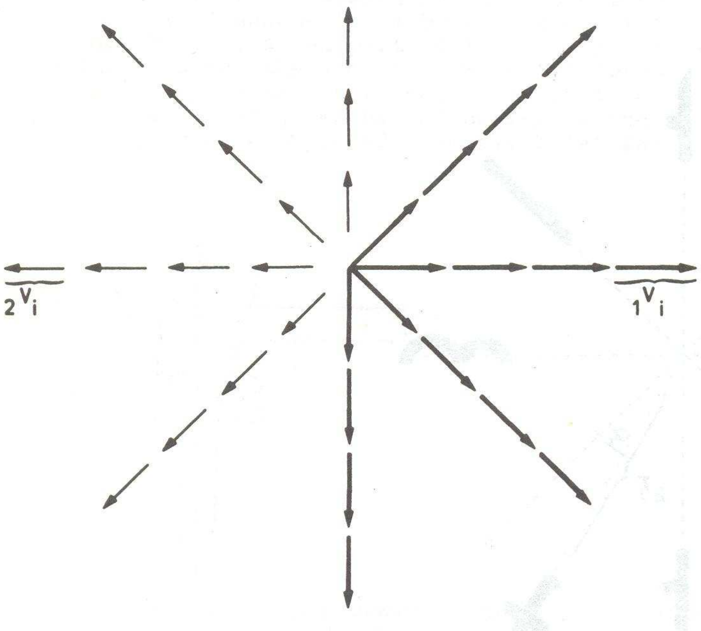

Fig. 9: Example of dynamic centrosymmetric pattern. The arrows indicate the velocities. The symbols refer to index S_{Cd}.

dynamic centrosymmetric patterns is:

$$S_{Cd} = \frac{\sum\limits_{i=1}^{i=n} \frac{{}_1v_{r_i,l_i}}{{}_2v_{r_i,l_i}}}{n} \tag{12}$$

The index provides a simple invariant for the detection of moving objects: Any object into which radii can be inscribed with $S_{Cd} = 0$ moves on a colliding path with the observer or the observer approaches it head on.

The formulas developed so far are tailored for the case of central and bilateral symmetry; any other form of symmetry (e.g. translational, rotational, or combinations of them as, for instance, ornamental, or crystallographic symmetry, following the taxonomy of WEYL (1952), cannot be measured appropriately by this index. However, the group-theoretic analyses of these types of symmetry allows for a generalization of S_M to these cases too.

Assuming that the structurally simplest form of symmetry is bilateral (as, for instance, WEYL (1952) assumes) with two possible forms of violation of symmetry (see Figure 7), it is possible to define the possible forms of violation for the other kinds of symmetry. Figure 1o illustrates

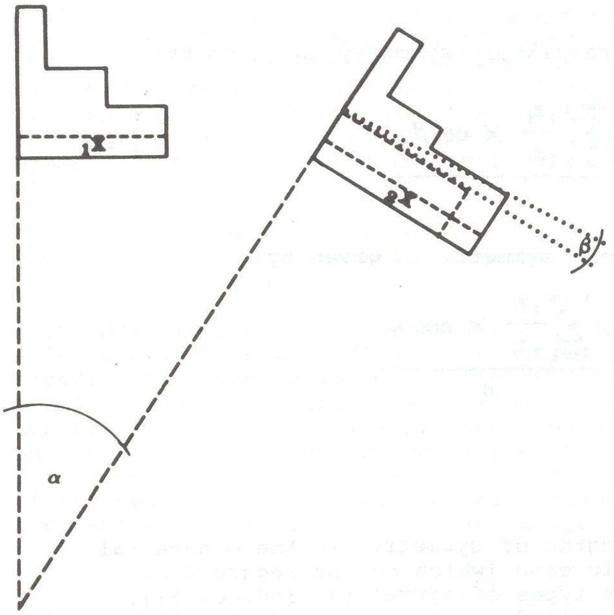

Fig. 1o: A form being nearly symmetric according to the translatory type of symmetry. The symbols refer to the index for translatory symmetry, S_T.

the case of translatory symmetry and Figure 11 illustrates the case of rotational symmetry. The index for the degree of translatory symmetry is given by:

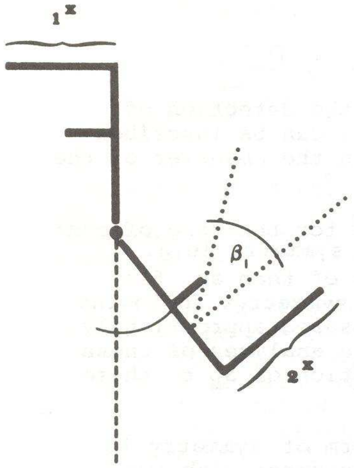

Fig.11: A form being nearly symmetric according to the rotational type of symmetry. The symbols refer to the index for rotational symmetry, S_R.

$$S_T = \frac{1-\cos\alpha}{2} \times \frac{\sum_{i=1}^{i=n} \frac{{}_1x_i}{{}_2x_i} \times \cos\beta_i}{n} \qquad (13)$$

and the index for rotational symmetry is given by:

$$S_R = \frac{1+\cos\alpha}{2} \times \frac{\sum_{i=1}^{i=n} \frac{{}_1x_i}{{}_2x_i} \times \cos\beta_i}{n}. \qquad (14)$$

In order to measure the degree of symmetry in the ornamental and in the crystallographic case (which can be regarded as mixtures of the first four types of symmetry) indices S_{11}, S_C, S_T and S_R have to be applied successively and the index with the highest absolute value should be taken as the appropriate metric of symmetry.

In an application of this kind of metric analysis to the nearly symmetric stimuli in the FREYD and TVERSKY study the problem of defining a proper perceptual unit was solved by imposing a general above-threshold grid on all forms. Then the filled-in grid points were counted and the numerical values entered into indes S_{11}. A comparison of the symmetry-measures for 64 forms with symmetry-ratings by subjects revealed a nearly perfect correlation between the subjective and the metric evaluation of bilateral symmetry.

However, FREYD and TVERSKY's Experiment III on decision times for the recognition of forms in the presence of more or less alternatives, reveals why this measure can only work well if the impression of symmetry relies mainly on features on an intermediary level. The reason for this restriction is the problem that there is no general rule for determining the structural units of the forms under investigation. Syntactic pattern recognition (FU 1974) seems to provide an alternative approach for developing indices for structural characteristics of forms as, for instance, symmetry. The measurement of figural complexity by LEEUWENBERG's (1978) system of figural analysis which follows these ideas in a way provides a means for including global characteristics as well as local characteristics, but it is bound to lead to an infinite regress - at least in some cases - since it relies partially on the notion of symmetry itself.

In order to overcome the restrictions of the proposed indices for measuring the degrees of symmetry it is necessary to embed them into a more general framework which takes into account the saliency of different forms of symmetry, the orientation of symmetric patterns in space, and the different levels of analysis in symmetry perception. The definition of different types of symmetry by means of the number of reflection planes necessary to generate them suggests that translatory and rotational symmetry should be less salient than bilateral symmetry. Experimental results (e.g. PALMER and HEMENWAY 1978) for the difference in detection times for bilaterally and rotationally symmetric forms support this hypothesis. Regarding the importance of orientation for symmetry MACH (1871/1903) states:

> "The vertical symmetry is aesthetically pleasing, whereas the horizontal symmetry is of no importance and is detected only by the experienced eye" (p. 105, my translation)

Results from detection-time experiments (e.g. PALMER and HEMENWAY 1978) are in line with MACH's conjecture. One problem with these experimental results is that either the forms have been perfectly symmetric or the lack of symmetry has been determined arbitrarily. In the following experiments the developed indices have been used to measure the degree of symmetry in different forms. On the basis of this symmetry metric the joint effects of the degree of symmetry, the orientation of the forms, and the type of symmetry have been investigated.

4. Experiment 1

In order to investigate the influence of the orientation of a form on the detectability of its symmetry it is necessary to determine if this influence is of an all-or-none type, as MACH apparently implies, or if it is continous. On each trial, subjects were presented with a stimulus which was either perfectly symmetric or very low in symmetry ($S_M \approx 0{,}5$). The orientation of this axis of symmetry was changed at random between 0° (horizontal orientation) and 90° (vertical orientation). Subjects were asked to decide as fast as possible if a presented stimulus was symmetric or not.

4.1. Method

4.1.1. Subjects

Twelve undergraduate students (at Westfälische Wilhelms-Universität, Münster) received course credit for participating in this experiment. The approximate time in the experiment for each subject was about 1 hour and a half.

4.1.2. Stimuli

Ten pairs of stimuli (one perfectly symmetric stimulus and the other low in symmetry but similiar in its overall form to the symmetric stimulus) were drawn. All drawings were simple closed, abstract, rectangular polygons similar to the stimuli in the FREYD and TVERSKY Experiment 1 (see Figure 12)

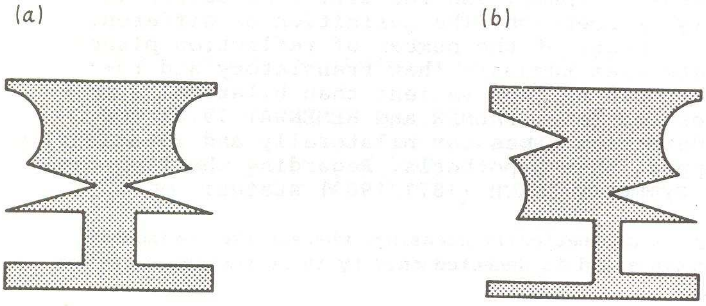

Fig. 12: A pair of stimuli as used in Experiment 1. (a) perfectly symmetric (b) low in symmetry but similar in form.

4.1.3. Procedure

The experiments were run individually. The stimuli were shown in a tachistoscope for as long a time as the subjects needed for making their decisions. When the subjects had decided whether the stimulus presented was symmetric or not they pushed a button. Pushing the button removed the stimulus and replaced it by a random grid for 4 seconds. Then the next stimulus was presented. The trials were blocked. In each block one pair of stimuli was presented in 19 different orientations (from 0° to 90° in steps of 5°). In each block the symmetric stimulus was shown ten times and the non-symmetric stimulus nine time. The serial order of the stimuli and the orientations was randomized. It took the subjects about 5 minutes to perform one block. A total of 10 blocks was run with each subjects. For 6 subjects the right-hand button was for the symmetric stimuli and the left-hand button for the unsymmetric stimuli. For the other subjects this was reversed in order to cancel out handedness effects.

4.2. Results

The average percentage of errors was less then 5% for all subjects and blocks. For this reason only the correct detections are analyzed. The data for the unsymmetric stimuli are disregarded too, because nothing can be said about their axis of symmetry. As predicted, there was a significant difference between the detection times for symmetric forms in either horizontal or vertical orientation for all subjects (see Table 1 for individual results).

Table 1

Subject number	Identification times (ms)	
	Horizontal stimuli	Vertical stimuli
1	875	832
2	893	844
3	883	834
4	889	847
5	897	849
6	941	886
7	910	859
8	892	848
9	916	868
10	909	857
11	922	869
12	939	892

The average detection times, however, reveal that the orientation effect is continously dependent on the angle of orientation (see Figure 13). The marked dip for an orientation of 45° differs significantly from the maxima of the function at about 25° and 65° ($t(19) = 2.979$, $p(\alpha) << 0.01$).

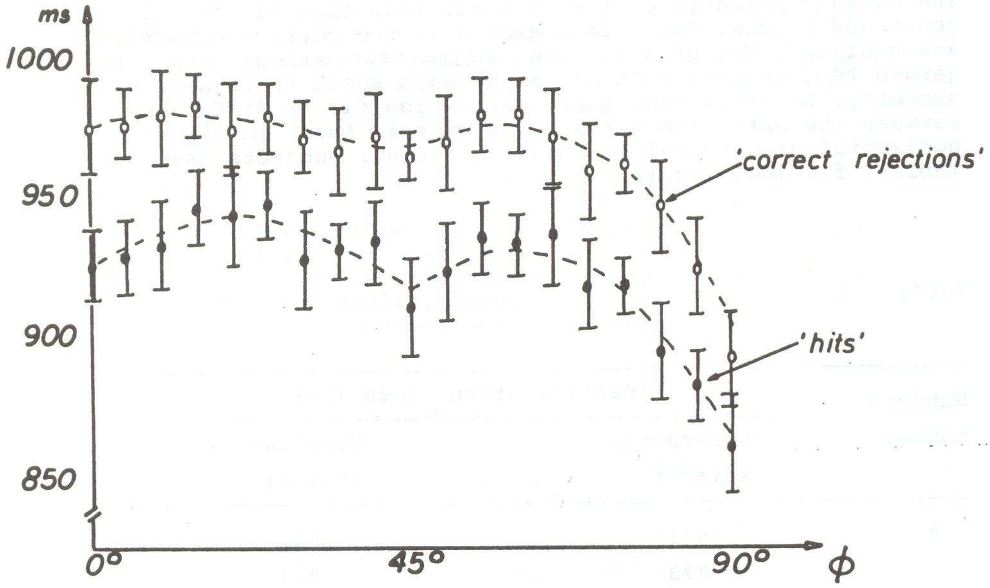

Fig. 13: Detection times as a function of angular orientation. Filled circles indicate averages and bars indicate standard deviations.

4.3. Discussion

The results clearly indicate that the influence of orientation on the detection of symmetry is continous with minima at the most 'natural' orientations. The strong influence of the vertical orientation is in line with other experimental results (e.g. PALMER and HEMENWAY 1978). As argued above, it can be assumed that the importance of bilateral vertical symmetry for the orientation in space is the cause for this superiority.

There are, at least, two ways the orientation could influence the perception of symmetry: (i) The perceiver imposes the vertical vs. horizontal frame of reference on the stimuli; or (ii) the perceived visual scene including the stimuli provides a frame of reference.

5. Experiment 2

Experiment 1 does not discriminate between the two possibilities of either a general frame of reference or a relative frame of reference. Experiment 2 was designed to investigate this question.

In this experiment subjects had to perform the same task as in Experiment 1. The stimuli were identical to the stimuli in Experiment 1 except for black rectangular frames surrounding the stimuli in the same orientation as the axis of symmetry. If the visual scene as a whole determines the frame of reference which influences the detection of symmetry then the black frames should offset the orientation effect observed in Experiment 1. If, however, the perceiver imposes his or her frame of reference upon the stimuli a result similar to Figure 13 is to be expected.

5.1. *Method*

5.1.1. *Subjects*

Fifteen undergraduate students (at Westfälische Wilhelms-Universität Münster) received course credit for participating in this experiment. Subjects took about 1 and a half hours to complete the experiment.

5.1.2. *Stimuli*

The same stimuli as in Experiment 1 were used in Experiment 2 except for the added black frames in the orientation of the axes of symmetry (see Figure 14).

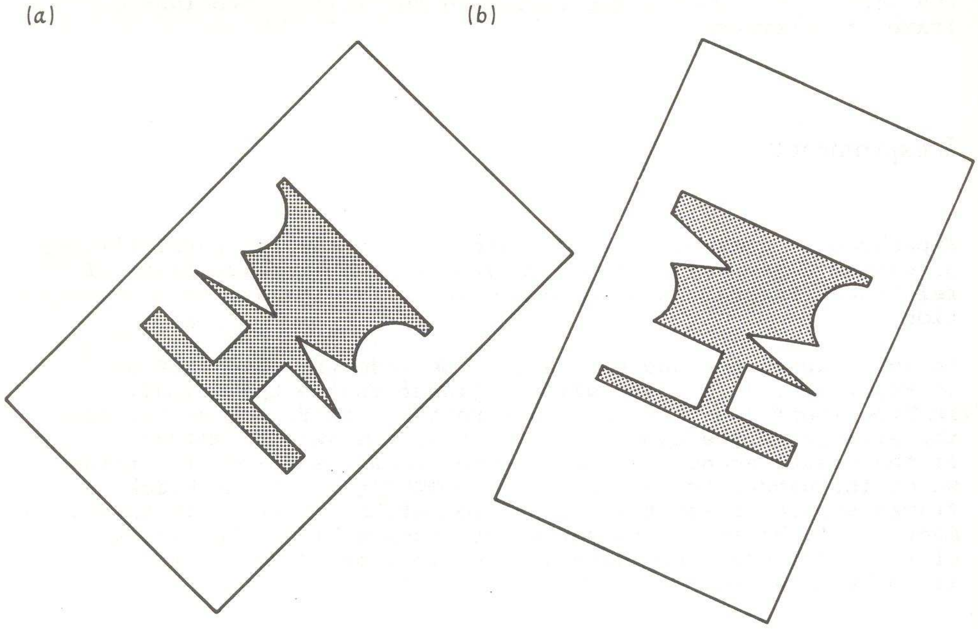

Fig. 14: A pair of stimuli as used in Experiment 2. (a) A perfectly symmetric stimulus in an orientation of 45°; (b) a stimulus low in symmetry in an orientation of 65°.

5.1.3. Procedure

The procedure was the same as in Experiment 1, except for an apparatus which held the orientation of the head constant.

5.2. Results

There are two major results: First, the introduction of a frame surrounding the stimulus in the same orientation as the axis of symmetry speeds up the detection significantly (t-test for correlated samples (orientations), $t(18) = 3.187$, $p (\alpha) << 0.001$). Second, both the relative and the general frame of reference influence the orientation effect. Between 0° and 55° the general frame of reference prevails, whereas from 60° upwards the orientation effect is offset by the relative frame of reference (see Figure 15).

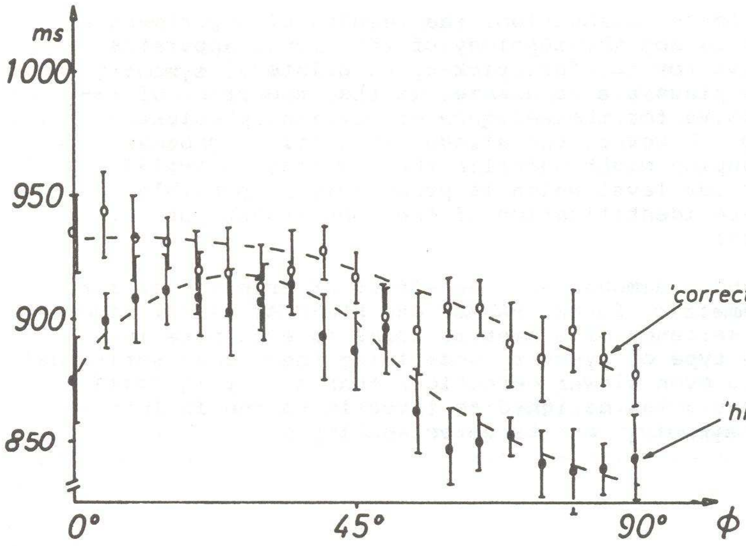

Fig. 15: Detection times as a function of angular orientation. Filled circles indicate averages and bars indicate standard deviations.

5.3. *Discussion*

The results clearly indicate that in a general upright position the relative frame for orientational reference is even more important than the general frame of reference provided by the visual apparatus. If this result can be generalized then the neurological explanation for the prevalent saliency of vertical bilateral symmetry can be ruled out. This explanation has been proposed by MACH (1897) and has gained renewed actuality by the work of JULESZ (1971) and CORBALLIS (1976). MACH (p. 53) states:

> "In a figure symmetrical with respect to the median plane, similar space sensations corresponding to the symmetric directions take the place of identical space relations. The right half on the figure stands in the same relation to the right half of the visual apparatus as the left half of the figure to the left part of the visual apparatus".

CORBALLIS (1976, p. 82) summarizes the consequences of MACH's position and the subsequent work of JULESZ (1971):

> "...its appears that there may be at least two different kinds of neural mapping involved in the perceptual integration across the vertical meridian. One is a process which preserves left-right orientation and which serves to maintain perceptual continuity across the visual field. The other is a more primitive possibly subcortical process which involves homotopic mapping and which enables us to rapidly detect left-right symmetry.

In contrast to these explanation, the results of Experiment 2 suggest that it is not the topology of the visual apparatus which is decisive for the fast pick-up of bilateral symmetry What seems more plausible to assume, is that the frame of reference is decisive for the salience of vertically oriented mirror symmetry · However, the alleged subcortical process of homotopic mapping might underlie the symmetry perception on the subconscious level which is presumably responsible for the immediate identification of the constituents of a form (BRADY, in press).

Experiments 1 and 2 demonstrate the effect of orientation for bilaterally symmetric forms. PALMER and HEMENWAY (1978) have shown that the saliency of symmetric forms is even more influenced by the type of symmetry underlying the forms: Rotational symmetry led to even slower detections than nearly symmetric forms. Experiment 3 was designed to investigate the influence of the type of symmetry on its detectability.

6. Experiment 3

The hypothesis underlying this experiment can best be captured by two different definitions of symmetry underlying the research on symmetry:

(i) "The pleasing effect of symmetry is caused by the repetition of sensations" (MACH 1871/1903, p. 102, my translation).

(ii) "The label 'symmetrical' is applied to every (finite or infinite) figure which may be made to coincide with itself by one or several successive reflections in planes" (SHUBNIKOV and KOPTSIK 1974, p. 127).

Definition (i) implies that the repetition of elements according to a rule should yield the most salient symmetric forms. Such a repetition underlies translatory symmetry. The next salient type of symmetry should by rotational symmetry because here the constituents of the form are congruent and differ only in orientation. Least salient should be bilateral symmetry there is no repetition of elements, at least, if the formative rule for the repetition is confined to the plane.

Definition (ii) implies the prediction of highest saliency for bilateral symmetry because this type necessitates least reflections. The saliency of the other forms of symmetry should be less and depend on the number of reflections underlying their formative rules.

Already PASCAL in the *Pensées* implies that there is a relation between the saliency of a symmetric form and the speed of processing:

"Symmetry is what we see at a glance" (1658/1950, p. 491).

In order to test the predictions made on the basis of the two definitions, Experiment 3 was run following procedure of Experiment 1. Ten pairs of stimuli for translatory symmetry and ten for rotational symmetry have been tested, again the stimuli were either perfectly symmetric or symmetric to a degree of $S_R \approx 0.5$ or $S_r \approx 0.5$.

6.1. Method

6.1.1. Subjects

Fifteen undergraduate students (at Westfälische Wilhelms-Universität Münster) received course credit for participating in this experiment. The approximate time in the experiment for each subject was about 3 hours.

6.1.2. Stimuli

Two times 10 pairs of stimuli (one perfectly symmetric stimulus and the other low in symmetry but similar in its overall form to the other stimulus) were drawn. All drawings were simple, closed, abstract, rectangular polygons of the same degree of complexity as the stimuli in Experiment 1 (see Figure 16). Complexity was operationally defined as the number of subparts (e.g. bars and angles) of the form under investigation.

6.1.3. Procedure

The same procedure as in Experiment 1 was used.

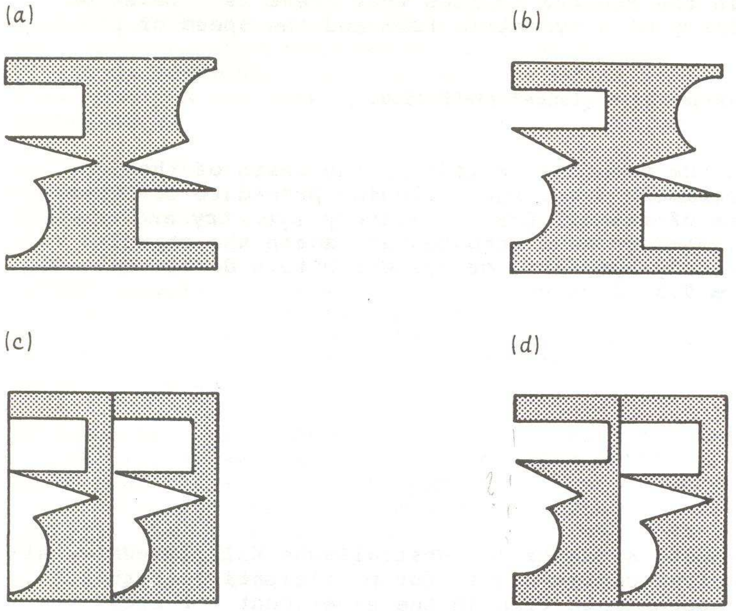

Fig. 16: Two pairs of stimuli as used in Experiment 3. (a) perfectly rotationally symmetric, (b) low in rotational symmetry but similar in form to (a), (c) perfectly translatory symmetric (d) low in translatory symmetry but similar in form to (c).

6.2. Results

The average percentage of errors was less than 8% for all subjects and blocks. For this reason only the correct detections are analyzed. The data for the unsymmetric stimuli are disregarded too, because nothing can be said about their axis of symmetry.

The comparison with the data from Experiment 1 shows that both translatory and rotational symmetry induce significantly longer detection times than bilateral symmetry ($t(19) = 3.017$, $p(\alpha) << 0.01$). However, this difference is mainly due to the strong difference between, on the one hand, translatory and rotational symmetry and, on the other, vertically bilateral symmetry (translatory vs. bilateral: $t(11) = 2.501$, $p(\alpha) < 0.05$; rotational vs. bilateral: ($t(11) = 2.243$, $p(\alpha) < 0.05$). None of the differences for the other orientations is significant but most of the differences point in the direction of a superiority of bilateral symmetry (see Figure 17).

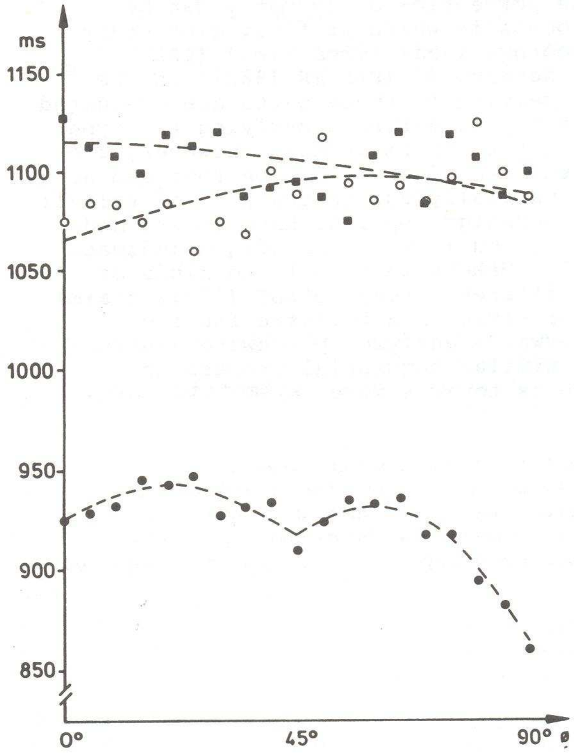

Fig. 17: Detection times for different types of symmetry as functions of angular orientation. Filled circles (bilateral symmetry), open circles (translatory symmetry) and squares (rotational symmetry), indicate averages.

6.3. *Discussion*

The results indicate clearly that phenomenal symmetry is not captured by the notion of symmetry as a repetition of similar or equal elements. Translatory and rotational symmetry take longer to detect and it can therefore be concluded that either different and more time consuming process than in the case of bilateral symmetry are necessary for the detection of these types of symmetry in forms.

In contrast to bilateral symmetry the orientation of the form seems to play a negligible role in the perception of these types of symmetry. The graphs of detection time as a function of angular orientation are essentially flat for both translatory and rotational symmetry (see Figure 17).

The experimental results obtained so far indicate that the proposed approach to measure the amount of symmetry works well, provided that the indices are weighted for saliency due to the type of symmetry or the orientation of the axis of symmetry. The underlying model for the perception of symmetry can be described by a two-stage process in which at first elementary forms are identified by 'smoothed local symmetries' (BRADY 1982) or 'spatial symmetry detectors' (PALMER 1982). On the second level the symmetries relying on these parts are evaluated by taking into account the group structure underlying the type of symmetry characteristic for the form at hand. However, the result that the process on the second level can be regarded as unitary might be, at least, partially due to the kind of stimuli chosen and the experimental paradigm applied. Experiments and observations using different stimuli and/or paradigms indicate that on the 'conscious level' (MARCEL in press) two kinds of symmetry perception can be differentiated: JULESZ (1971) claims that at first symmetry is perceived in a holistic fashion ('perception') and only afterwards analyzed piecewise ('scrutiny'). FREYD and TVERSKY propose a similar sequential process to account for the asymmetric bias towards more symmetric forms in their experiments:

> "If the form is perceived as symmetric, the perceiver takes advantage of that to make inferences about those parts of the form that he does not bother to inspect. ...Distortions may occur, when unbeknownst to the perceiver, the form was not as symmetric as the global impression suggested. This error at the global level may lead to misapplied inferences at the local level" (p.4).

7. Experiment 4

One way to test this model of multiple levels in the conscious perception of symmetry consists in using stimuli which are built up from symmetries on different levels, from entirely local symmetries to the global symmetry which in a way binds together the whole form. Examples for such forms can be found in decorative art (e.g. GOMBRICH 1979). In these forms violations of symmetry can occur on the different levels. This experiment was designed to question the assumption of a unitary process of symmetry perception. The comparison of ornaments in decorative art indicates that symmetry perception of complex forms might be further influenced by the way the local symmetries are integrated into the global symmetry. Some forms (e.g. windows in Gothic art) are built up by a simple inclusion operation, whereas other forms (e.g. tile work in Islamic art) are constructed on the basis of interlace which produces interconnected symmetries on different levels. It can be assumed that in highly integrated forms (e.g. built up by means of interlace) it is more difficult to detect local violations of symmetries than it is in less integrated forms (e.g. built up by means of the inclusion operation).

On each trial, subjects were presented with a stimulus which was either perfectly symmetric or in which symmetry was violated. Violations of symmetry was violated. Violations of symmetry occurred either on the topmost level of symmetry or on the most local level. The stimuli were either highly integrated or low in integration. Subjects were asked to decide as fast as possible if a presented stimulus was symmetric or not.

7.1. Method

7.1.1. Subjects

Fourteen undergratuate students (at Westfälische Wilhelms-Universität Münster) received course credit for participating in this experiment. The approximate time in the experiment for each subject was about 1 hour and a half.

7.1.2. Stimuli

Two times 10 pairs of stimuli (one perfectly symmetric stimulus and the other exhibiting one violation of symmetry either on the top-most level or on the most local level). One set of pairs of stimuli consisted of different drawings of Gothic church windows (approximately equal in complexity).

The other set consisted of drawing of Islamic tile work. Both sets were about comparable in the number of distinguishable subparts. The orientation of all stimuli was both vertical and horizontal (Figure 18).

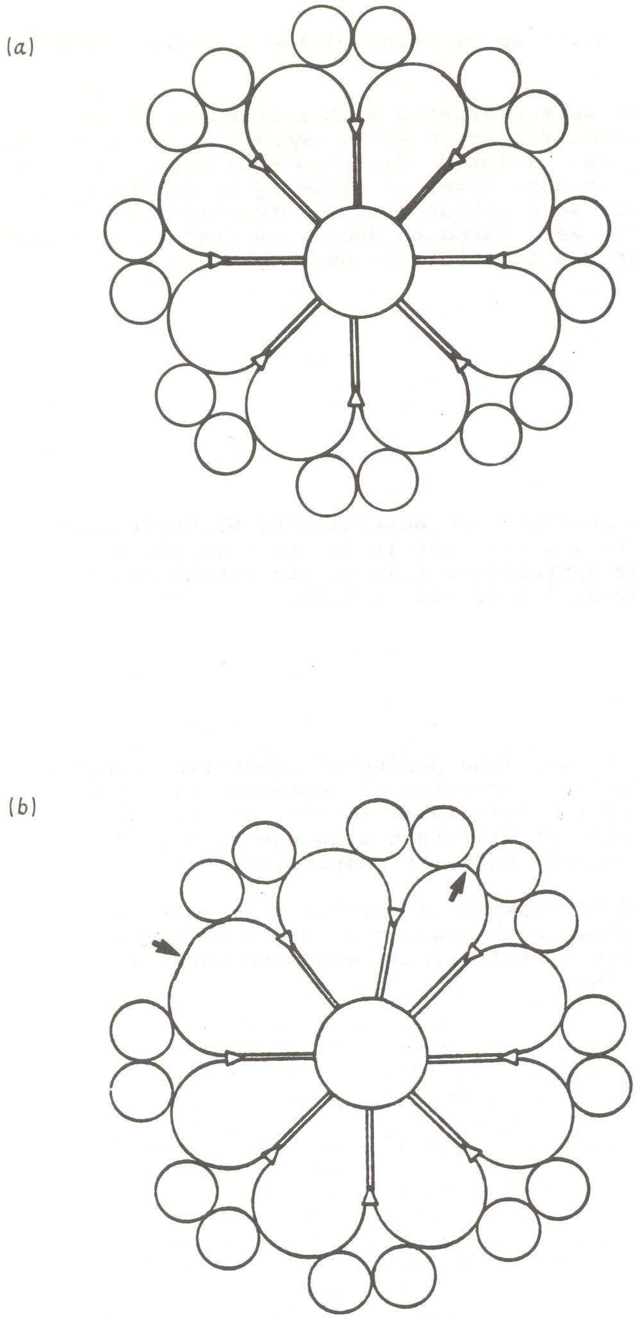

Fig. 18: a, b,stimuli as used in Experiment 4.(a) a perfectly symmetric Romanic window (b) the same with one global violation of symmetry;

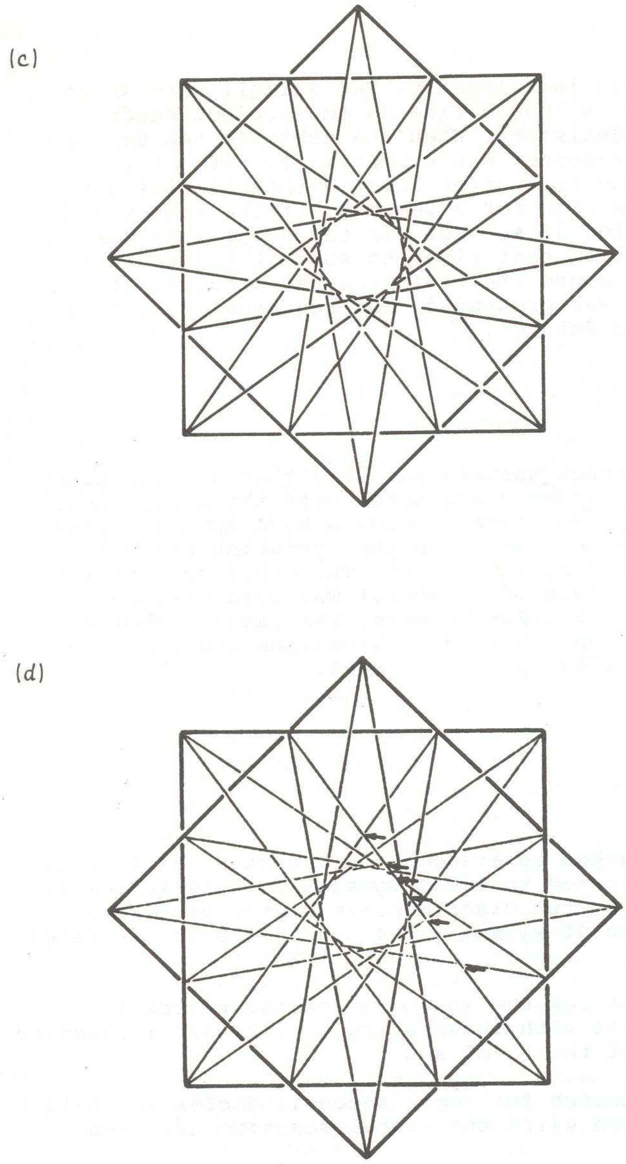

Fig. 18: c, d, (c) a perfectly symmetric piece of Islamic tile work (Alhambra)
(d) the same with a local violation of symmetry.

7.1.3. Procedure

The experiments were run individually. The stimuli were shown in a tachistoscope for as long a time as the subject needed for making his or her decisions. When the subjects had decided whether the stimulus presented was symmetric or not, they pushed a button. Pushing the button removed the stimulus and replaced it by a random grid for 3 seconds, if the answer had been 'symmetric', and for 15 seconds, if the subject had observed a violation. During that time the subject had to mark in a coordinate system where the violation had occurred after that the next stimulus was presented. The sequence of stimuli was randomized for each subject.

7.2. Results

The percentage of incorrect answers was less than 4%. The wrong answers were apparently randomly scattered over the conditions. For these reasons, only the correct answers have been analyzed. In general subjects reacted faster to the inclusion stimuli (Gothic windows): $t(19) = p$, $(\alpha) < 0.05$. The other main effect (global vs. local violations of symmetry) was even stronger $t(19) = 3.497$, $p\ (\alpha) \ll 0.01$. Furthermore, the results show a strong interaction between the kind of violation and the type of stimuli: $F\ (1;1) = 4{,}793$, $p\ (\alpha) < 0.01$.

7.3. Discussion

The results reveal a marked superiority in detecting violations of global symmetry compared to local symmetry. This indicates that there are, at least, two discriminable higher processes governing the perception of symmetry (4) on the conscious level (MARCEL in press):

(i) a global scanning for the symmetry of the entire form which is, at least with these stimuli, strongly influenced by the contour of the form, and

(ii) an intentional search for local inconsistencies which is apparently started *after* the global scanning has been terminated.

This interpretation is further supported by the strong interaction effect. The degree to which the symmetries in a form are integrated strongly influences the detectability of local violations of symmetry. The fact that the interaction is stronger than the

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[4] 'Higher' is understood here as compared to the more or less automatic detection of symmetry which underlies the identification of basic of forms.

main effect for 'type of stimuli' rules out a seemingly plausible alternative explanation for differences in detection times. Namely, that the ease with which subjects made their decisions concerning the symmetry in the Gothic-window stimuli is due to the fact that they are better accustomed to this kind of patterns.

8. General Discussion

In conclusion: The theoretical as well as experimental results support the definition of symmetry as a characteristic of forms generated by reflections in one or more planes. The phenomenological saliency of symmetry depends on its type, that is, the more reflections are necessary to generate a symmetric form the less salient the symmetry appears. For the most basic, the bilateral, symmetry the orientation of the plan of symmetry influences the saliency further. It seems plausible to assume that this is due to the importance of detecting vertical symmetry for the orientation in space. The process of perceiving symmetry can roughly be classified into three subprocesses:

(i) Detecting 'elementary' forms by 'spatial analyzers' (PALMER 1982) or 'smoothed local symmetries' (BRADY 1982). This subprocess is presumably unconscious and automatic.

(ii) Picking-up the global symmetry of a form. LOCHER and NODINE's (1973) results together with the results of Experiment 1 - 3 indicate that only vertically bilateral symmetry is automatically perceived the way it is described by CORBALLIS (1976): "We may know that a shape is symmetrical before we know what else it is" (p. 77).

(iii) Scrutinizing a given form for structural rules, especially for different types of symmetry: bilateral symmetry (either locally or globally in a non-horizontal orientation), rotational, and translatory symmetry.

The apparent importance of bilateral symmetry in perception poses the question why this is the case: Is it that they world tends to bilaterally symmetric forms or is it that we are tuned to impose bilateral symmetry upon the variety of forms in the environment? The 'Critical Realist' position, especially as expressed in SHEPARD's (1981) principle of 'psychophysical complementarity' provides a tentative answer by reformulating the problem and by indicating additional directions of research.

(i) There is symmetry in the physical world as, for instance, expressed by the tendency towards stable states which are characterized by balance and which require minimal work to be conserved.

(ii) The constraints due to the symmetry in the physical world are the reason for the high survival value of bilaterally symmetric forms in biology. This in turn has made the world more symmetric, that is, the environment for our perception.

(iii) In an environment where symmetry is a major structural constituent, the fast pick-up of symmetries is of high survival value for the perceiver.

(iv) The interplay of these constraints and the constraints due to 'shareability' (FREYD, in press) namely that we live in a world where we have to share our knowledge about it, influences the way we describe and manipulate this world. This in turn leads to the emergence of more symmetries underlying physical processes (see HALL 1969; BOARDMAN 1973).

Especially the point (iv) makes apparent why the alternative Rationalist point of view ('humans impose symmetry upon the world' and 'worlds are created by mind and perception'(cf. GOODMAN 1978; WHEELER 1979)seems to plausible in a world characterized by technology and science. However, the experimental results indicate that this is only one side of the coin.

Point (iii) suggests experiments on symmetric forms in settings where picking up symmetries is of high importance for the actions of the perceiver. For the regulation or detection of movement in space bilateral and central symmetry provide invariants which makes possible the detection of objects moving on colliding paths (BALL and TRONICK 1971) or the determination of one's own direction of movement in relation to other stationary or moving objects. For these reasons I plan to do experiments in which the symmetric patterns and/or the perceiver are moving.

Summary

Symmetry is defined as the balanced proportion of parts in a whole. The main types of symmetry are bilateral, translatory, rotational, and central symmetry which are important for different perceptual tasks and situations. It is argued that detecting the type of symmetry characteristic for a form at hand is important for the identification of this form. The failure of most pattern recognition algorithms to pick up this kind of information is seen as one of the reasons for the apparent differences in the performance of human and computer vision. The measurement of the degree of symmetry in a form is a necessary precondition for the interpretation of behavioral data about form perception in general. Different approaches, stemming from information theory, autocorrelation theory, and proximity analysis, are critically examined and shown to be either insensitive to even perceptually striking instances of symmetry, or to be dependent on a-priori frames of reference or a special type of symmetry. An alternative measurement approach is suggested which relies on WOLF's (1949) and WEYL's (1952) group-theoretic analyses of symmetry. Formulas are developed that measure how much the constraints derivable from the group-theoretic characterizations of symmetry are met by specific forms. The applicability of these measures is shown for forms differing in the degree of symmetry (FREYD & TVERSKY). In a series of experiments I investigate how much the general or relative orientation of the form influences the perceptual saliency of symmetry (Experiments 1 and 2) and how much this depends on the type of symmetry (Experiment 3). I find that symmetry along the vertical axis is more salient than symmetry along the horizontal axis for bilaterally symmetric forms. Frames giving rise to relative orientation partially offset this effect. In a final experiment (4) I try to determine whether symmetry information is processed on different levels. A three-level model is suggested and its implications for theoretical and empirical questions concerning symmetry are discussed.

Zusammenfassung

Symmetrie wird definiert als das ausgeglichene Verhältnis von Teilen in einem Ganzen. Die Hauptformen der Symmetrie sind Spiegelsymmetrie, Wiederholungssymmetrie, Drehsymmetrie und Zentralsymmetrie; diese verschiedenen Arten der Symmetrie sind für unterschiedliche Wahrnehmungsaufgaben und -situationen von Bedeutung. Es wird argumentiert, daß die Entdeckung des Symmetrietyps, der für eine bestimmte Form charakteristisch ist, für die Identifikation dieser Form entscheidend ist. Das Versagen der meisten Mustererkennungsalgorithmen, diese Art von Information zu verarbeiten, wird als einer der Gründe für die offensichtlichen Unterschiede zwischen menschlicher Wahrnehmung und "Computervision" angesehen. Die Messung des Grades von Symmetrie in einer gegebenen Form ist eine notwendige Vorbedingung für die Interpretation von Verhaltensdaten über die Formwahrnehmung. Unterschiedliche Zugänge zu diesem Problem (aus der Informationstheorie, der Autokorrelationstheorie und der Proximityanalyse) werden kritisch untersucht und es wird gezeigt, daß sie entweder unempfindlich gegen viele "ins Auge fallende" Symmetrieformen sind, oder von vorher gewählten Bezugsrahmen bzw. Auflösungsrastern abhängen. Es wird vorgeschlagen, an das Problem der Messung aus der Tradition gruppentheoretischer Untersuchungen der Symmetrie (WOLF 1959 und WEYL 1952 heranzugehen. Es werden Maßformeln entwickelt, die angeben, in welchem Umfang die Bedingungen von bestimmten Symmetrieformen durch eine vorliegende Form erfüllt sind. Die Anwendbarkeit dieser Maße zeigt sich bei Formen, die sich hinsichtlich ihres Symmetriegrades unterscheiden (FREYD & TVERSKY 1983). In einer Reihe von Experimenten wurde auf diesem Hintergrund untersucht, welchen Einfluß die relative oder absolute Orientierung einer Form für die Entdeckung von Symmetrie hat (Experiment 1 und 2) und wie sehr dies von der vorliegenden Symmetrieart abhängt (Experiment 3). In spiegelsymmetrischen Formen führt eine vertikale Orientierung der Spiegelungsachse zu signifikant kürzeren Entdeckungszeiten als eine horizontale Orientierung. Benutzt man Rahmen, um eine relative Orientierung der Form zu induzieren, wird dieser Effekt teilweise unterdrückt. In einem abschließenden Experiment (4) wird untersucht, ob Symmetrie auf verschiedenen Ebenen verarbeitet wird. Aufgrund der Ergebnisse wird ein Drei-Ebenen-Modell vorgeschlagen und es wird gezeigt, welche Konsequenzen für theoretische und empirische Fragen sich hinsichtlich Symmetrie aus diesem Modell ergeben.

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