QUANTIFICATION OF GESTALT LAWS AND PROPOSAL OF A PERCEPTUAL STATE-SPACE MODEL

Tadasu Oyama & Hisao Miyano

Introduction

Gestalt psychology has a strong influence on the present study of psychology, especially in the field of perception. Despite promising early attempts, such as Korte's set of quantitative rules describing apparent motion (see Sarris, 1989), the Gestalt laws are often understood and applied on a more qualitative level. To allow for preciseness and predictive power, these early attempts for quantification of perceptual laws should be revived, taking advantage of the present-day means of computation (see Kubovy & van der Berg, 2008). Without quantification, any exact prediction can hardly be drawn.

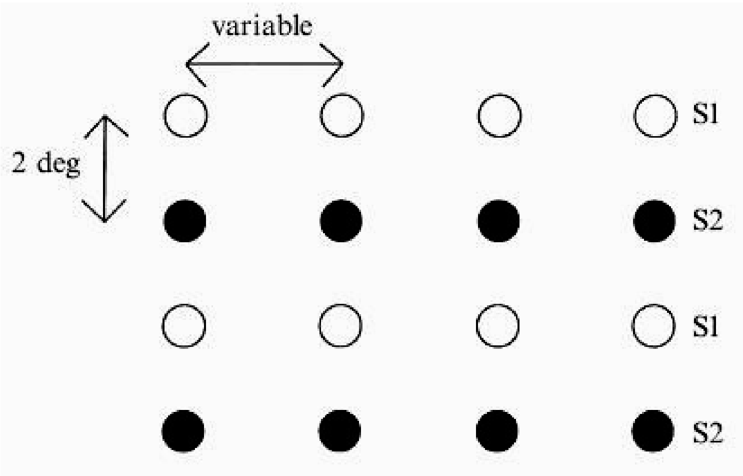

Figure 1. The stimulus pattern used in Oyama, Simizu & Tozawa (1999) experiment of perceptual grouping. After Oyama (1997). Copyrighted by Pion Ltd, London. Reproduced by permission.

If, for example, the law of proximity and the law of similarity work in different directions, vertical versus horizontal grouping, in a given situation such as in Figure 1, we cannot predict any exact final result from the traditional Gestalt laws, namely, which of the two factors, proximity or similarity, overcomes the other factor and determines the final results (Wertheimer, 1923).

Several studies have been conducted for quantification of these laws, especially for the proximity (see Kubovy, 1994; Kubovy, Holcombe, & Wagemans, 1998; Kubovy & Wagemans, 1995; Oyama, 1961). Hochberg & Silverstein (1956) and Hochberg & Hardy (1960) tried to compare the law of similarity with that of proximity quantitatively. A similar study by Oyama, Simizu and Tozawa (1999) will be continued here more systematically.

Quantification of Gestalt Laws

An attempt was made to quantify the main Gestalt laws on the basis of experimental results of Oyama et al. (1999) on two traditional perceptual problems in Gestalt psychology, perceptual grouping and apparent motion.

Matching between Proximity and Similarity Factors in Perceptual Grouping. In the experiment on perceptual grouping, Oyama et al. (1999) used a stimulus pattern as shown in Figure 1 and employed the trade-off strategy in the terminology of Kubovy et al. (2008). This pattern can be seen as four horizontal rows of homogeneous objects or four vertical columns of heterogeneous objects. The stimulus pattern was presented on the computer display for 3 seconds per trial. The observer responded to the joystick according to whether four horizontal rows or four vertical columns were seen. The horizontal separation between stimulus objects was varied in small steps (15 min. in visual angle) depending on the observer's responses, while the vertical separation was kept at 2 deg; after a "horizontal row" response, the horizontal separation was increased one step. After a "vertical column" response, the horizontal separation was reduced one step. A suitable combination of some larger horizontal separation between homogeneous objects and a smaller constant vertical separation between heterogeneous objects produced two kinds of perceptual grouping with equal probabilities. Such a matched horizontal separation was obtained with the double staircase method for each stimulus pattern.

Usually the obtained matched horizontal separation is greater than the constant vertical separation (2 deg.). In horizontal rows, the weaker proximity factor and the stronger similarity factor work together, and in vertical columns the stronger proximity factor and the weaker similarity factor work together. These two different pairs of the proximity and similarity factors matched each other in the obtained spatial relations. Such matched spatial relations were studied as a function of dissimilarity between the two kinds of stimulus objects.

In the stimulus patterns, one kind of object was always red small dark discs (original or standard objects). Another kind of object (paired objects) varied among 16 combinations of two hues (red and green), two brightness (dark and bright), two sizes (small and large) and two shapes (disc and equilateral triangle).

In general, the matched separation increased as the number of differences between the stimulus objects increased. Even when the number of differences was the same, the obtained matched separations were varied among the different perceptual dimensions, hue (H), brightness (B), size (S) and shape (SH). For example, in the conditions with single difference, the effect of hue or size difference on the matched separation is larger than the effect of brightness or shape difference.

Quantification of Similarity Factors across Various Perceptual Dimensions. To examine more systematically these variations among perceptual dimensions in the effects of dissimilarities on the matched separations, we conducted a multiple linear regression analysis on their data.

The obtained regression formula was as follows:

$$X = 2.04\,D + 0.43\,H + 0.20\,B + 0.39\,S + 0.36\,SH \quad (1),$$

where *X* represents the matched horizontal separation in visual angle, and *D*, *H*, *B*, *S* and *SH* are the vertical separation, hue, brightness, size, and shape, respectively. The values of these variables are 1 if the respective differences exist and 0 if no difference exists. The value of *D* is always 1, because the constant vertical separation always exists.

The regression coefficient on the vertical separation (*D*) is 2.04. It is nearly equal to the vertical separation in visual angle. The regression coefficient on the hue, 0.43 indicates that the effect of the hue difference between red and green corresponds, in its negative effect on perceptual grouping, to that of an increase of the horizontal separation of 0.43 deg. in visual angle. In the same way, the brightness difference between the dark and bright objects (0.3 and 0.8 cd/m^2) corresponds to an increase of 0.20 deg. in the spatial separation, the size difference between small and large objects (0.7 and 1.4 deg. in diameter of discs or corresponding size difference of triangles of the same areas as the discs) corresponds to an increase of 0.39 deg. in the spatial separation, and the shape difference between disc and triangle corresponds to an increase of 0.36 deg. in the spatial separation.

This multiple regression formula is very effective as shown by the high coefficient of determination (adjusted $R^2 = 0.830$). According to the obtained regression formula, we can even predict the results of a new stimulus pattern consisting of combinations among the above 16 variations of objects.

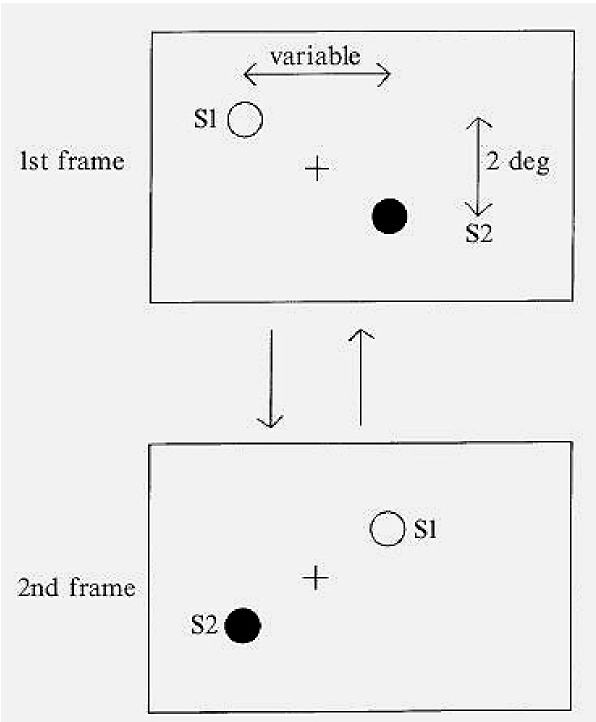

Figure 2. The stimulus pattern used in Oyama, Simizu & Tozawa (1999) experiment of apparent motion. After Oyama (1997). Copyrighted by Pion Ltd, London. Reproduced by permission.

Matching between Proximity and Similarity Factors in Apparent Motion. A similar experiment was also done on apparent motion. Two stimulus patterns as shown by the first and the second frames in Figure 2 were presented alternatively with 100 ms durations and 100 ms ISIs (inter-stimulus intervals) in the same area on a computer display. The first frame consisted of two different stimulus objects shown by the solid and open discs (e.g. red and green small discs) on the top left and the bottom right, while the second frame included the same pair of objects on the top right and the bottom left. One of two kinds of apparent shuttle motion can be seen in this stimulus situation: either horizontal apparent motions between the homogeneous stimulus objects or vertical apparent motions between heterogeneous stimulus objects. The vertical separation between the two heterogeneous objects was always 2 deg. in visual angle and the horizontal separation between the two homogeneous objects was varied by the observer's responses on the direction of apparent motion, in the same way as for the grouping experiment (see above).

Experimental results (matched horizontal separations) were obtained with the double staircase method, in the same 16 conditions of dissimilarities as those used in the grouping experiment. Similar results were obtained, however, the effects of dissimilarities look somewhat weaker than those in the grouping experiment.

The same technique of multiple-regression analysis was applied again, and it revealed the following formula:

$$X = 2.29\,D + 0.23\,H + 0.05\,B + 0.16\,S + 0.11\,SH \quad (2),$$

where the meanings of the variables are the same as those in formula (1). The regression coefficients for apparent motion were generally smaller than those in the grouping experiment, but their order of magnitude was exactly the same in the two experiments. The largest regression coefficient was that of hue, the second largest was that of size, the third largest was that of shape and smallest was that of brightness. This fact suggests an interesting stability of the relative effects of similarities in different perceptual dimensions across the two phenomena, the perceptual grouping and apparent motion, but it should be noted that these relative effects are limited to the specific stimulus variations chosen for the hue, brightness, size and shape dimensions employed in the two experiments.

This regression formula is very effective again as shown by such a high coefficient of determination (adjusted $R^2 = 0.967$).

Toward a Model

On the basis of the above-mentioned quantification of the Gestalt laws, we would like to propose a "Perceptual state-space model" and to discuss its relation to perceptual grouping, apparent motion and some other perceptual phenomena A "state space" in system science represents a space in which the state of a dynamical system is represented as a point, and "state" means any well defined condition or property that can be recognized if it occurs again (e.g., Ashby, 1956). In this section, we explain how this concept of state space can be used to describe the phenomena of perceptual grouping and apparent motion, although our state space approach is still at a preliminary level and cannot provide exact rules governing the dynamic behavior of perceptual state, as in system science.

Perceptual State Space. The perceptual state space in vision consists of several dimensions including perceptual dimensions of color (hue, brightness, and saturation), shape, size, as well as spatial dimensions (vertical, horizontal and depth). A point in the perceptual state space represents a perceptual object which has a certain color, a certain size and a certain shape, and is located at a certain point in the visual space of an observer. Different points in the perceptual state space correspond to different perceptual objects in some perceptual characteristics and/or in location. Distances between these points correspond to differences in perceptual characteristics and/or perceived spatial separation. Interactions between different points would indicate some perceptual phenomena related with these objects.

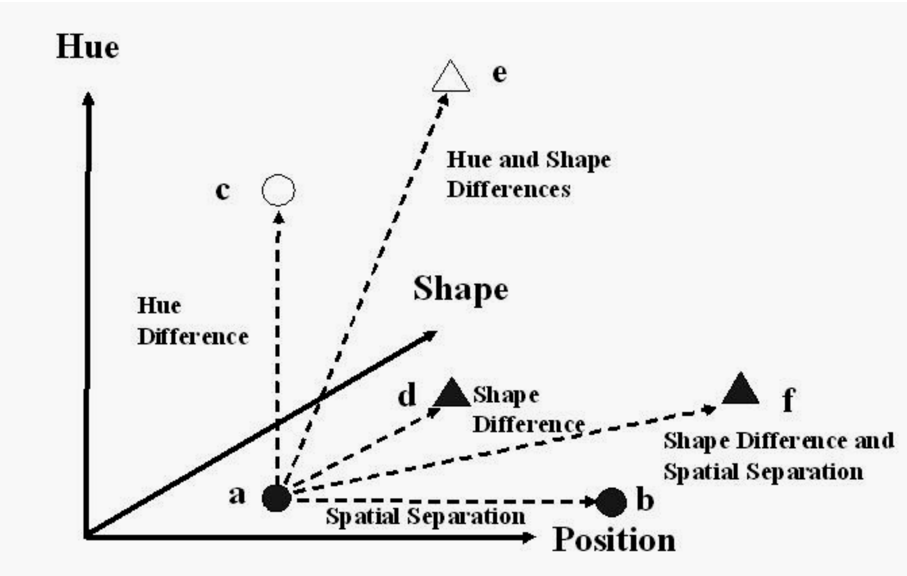

Figure 3. A schematic representation of the perceptual state space. Only three related coordinates (horizontal position, hue and shape) are shown in this figure for simplicity. Solid symbols indicate red objects and open symbols, green objects. Arrows represent perceptual differences and/or separations between objects.

In Figure 3, only three perceptual dimensions, namely hue, shape, and position in one of the spatial dimensions are shown, and other perceptual dimensions are omitted for simplicity. Shape is represented as one dimension in this figure, though it may have three or more dimensions as shown by Oyama, Miyano, & Yamada (2003), Zusne (1970) and others. Solid symbols indicate red objects and open ones indicate green objects. Hue is well known to be represented by polar-coordinates, but here it is schematically represented by a linear axis, again for simplicity. The point **a** corresponds to the red disc on the left side, the point **b** is a red disc on the right side in the visual field, the point **c** is a green disc on the left side and the point **d** is a red triangle on the left side. Three arrows from **a** to **b**, from **a** to **c**, and from **a** to **d** indicate the spatial separation between the left and the right red discs, the hue difference between the red and green discs, and the shape difference between the red disc and the red triangle, respectively.

The magnitude of differences or dissimilarities in hue, size and/or shape as well as spatial separations between different objects can be represented by distances between them in this perceptual state space, if all dimensions have a common scale. The multiple-regression formulae shown before give us a quantitative basis for estimating such a common scale among different perceptual dimensions. The obtained regression coefficient of each perceptual dimension will represent its relative value of dissimilarity as compared with a spatial separation in degrees of visual angle (the standard scale). If two objects are different in two perceptual dimensions, for example hue and shape, as a red disc and a green triangle, the combined dissimilarity is represented in such as the arrow from the solid disc **a** to the open triangle **e** in Figure 3. If two objects are different in more than two perceptual dimensions, for example shape and spatial position, as a red disc and a red triangle on different positions, the combined dissimilarity and separation is represented by the arrow from the solid disc **a** to the solid triangle **f** in Figure 3.

An experimental situation of perceptual grouping can be represented by arrows in the perceptual state space like Figure 3. In such a perceptual state space with the calibrated, adjusted common scale for every dimension, the length of each arrow represents the degree of dissimilarity and separation or the difficulty of perceptual grouping between the two different objects represented by the head and end of the arrow, respectively. The shortest arrow indicates the perceptual grouping which should occur most frequently. In this model, the spatial separation and the dissimilarities in hue, size and shape, and then the proximity and similarities are treated in the same way. *Thus the proximity factor and the similarity factor are synthesized or unified as a single factor, i.e. the proximity in the perceptual state space.*

Displacement from one point to another in this perceptual state space corresponds to a perceptual motion, if the direction of the displacement is related with the dimensions of the visual space. Such a displacement may also correspond to a perceptual change in hue, size or shape, if the direction of displacement in the perceptual state space is related with dimensions of hue, size or shape. Thus perceptual motions in the visual space and perceptual changes in hue, size and shape are represented here in the same way, except that the related dimensions are different between perceptual motions and other changes.

If a displacement is related with both the dimensions of the visual space and other perceptual dimensions of hue, size and/or shape as arrows like that one from **a** to **f**, perceptual motion and change(s) will occur at the same time.

Application to the Experimental Situations

The experimental situation of perceptual grouping shown by Figure 1 is represented in our model as shown in Figure 4, where the dimensions of the horizontal and vertical spatial positions and that of the hue of an object are represented, while the other perceptual dimensions are omitted for simplicity.

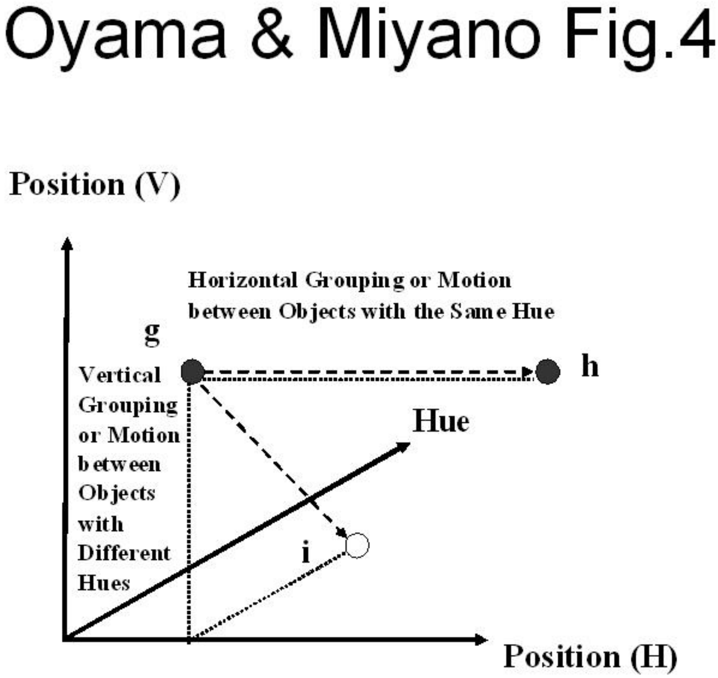

Figure 4. A schematic representation of the perceptual state indicating the experimental situation of shown in Figures 1 and 2. For simplicity only three related coordinates (horizontal and vertical positions and hue) are shown here. Solid symbols indicate red objects and an open symbol, a green object. Arrows represent possible perceptual groupings, apparent motions, or perceptual changes between these objects. Dotted lines indicate city-block distances and broken lines, Euclidean distances.

The point **g**, indicates a red disc in the stimulus pattern, while points **h** and **i** show red and green discs located in different positions in the same stimulus pattern, the right and lower points, respectively. In the perceptual state space shown in Figure 4, the arrows show possible groupings between two given objects and the vertical separation between the points **g** and **i** is smaller than the horizontal separation between the points **g** and **h**, but the points **g** and **i** are located in different depths. The three-dimensional separation, measured with the city-block or Euclidian metric as will be discussed below, between **g** and **h** can be matched to that between the points **g** and **i** in this perceptual state space. In such a situation, the probabilities of occurrence will be matched between the two possible groupings, a grouping of **g** and **h** and another grouping of **g** and **i**. For this prediction, the distance scaling should be calibrated and adjusted to be common in its function across the spatial dimensions and the hue dimension, as shown before. These relations should also hold across other dimensions such as brightness, size and shape which were omitted in the figure for simplicity.

The experimental situation of apparent motion shown in Figure 2 can also be represented in the perceptual state space in the same way as for perceptual grouping. In that case, arrows represent possible apparent motions and/or perceptual changes of one object from the first frame of the stimulus pattern to either object in the second frame. Our model shown in Figure 4 can be also used to predict which of these possible percepts occurs most frequently in a given ambiguous stimulus situation, if the relative lengths can be compared among the respective arrows. The perceptual motion or change or their combination represented by the shortest arrow is expected to occur most frequently, according to *the minimum principle or the principle of "least change"* (Hochberg, 1957; Johansson, 1958; Koffka, 1935; Metzger, 1953) applied to the perceptual state space. For this comparison between arrow lengths, a common distance scale should be defined across different dimensions, *i.e.*, spatial position, hue, size, shape and *etc.*, as discussed above.

It should be noted that the linear combinations of the effects of perceptual differences in different dimensions shown in formulae (1) and (2) indicate that the city-block distances (the sum of distances along each dimension as shown by dotted lines in Figure 4) rather than Euclidean distances (broken lines) determine probabilities of occurrence of perceptual grouping, apparent motion, and/or perceptual changes. We also tried to apply Euclidean metrics to the same results, and obtained slightly lower coefficients of determination (0.829 and 0.961 for grouping and apparent motion, respectively) than with the city-block metric.

According to the second author's analysis of unpublished data of Takashi Onuki who conducted a perceptual grouping experiment similar to Oyama et al. (1999) varying each of the hue, luminance, size and shape of the paired objects in three levels, the obtained coefficient of determination for the Euclidian metric was slightly larger than that for the city-block metric, but the difference was not statistically significant (adjusted $R^2 = 0.810$ and $R^2 = 0.790$ for the Euclidian and city–block metrics, respectively). Consequently, the present data do not firmly allow us to decide which metric is most suitable for the perceptual state-space model.

Further Possible Applications

Our model may also be applied to other perceptual phenomena, such as optical illusions, contrast, assimilation, esthetic harmony, balance, etc. These perceptual interactions between visual objects will be determined as a function of the distances between the points in the perceptual state space, in which distances represent perceptual differences or spatial separations between the interacting visual objects. Contrast and assimilation, which are familiar in brightness and color perception, are also found to occur in size, slant, curvature, depth, velocity and other perceptual dimensions, as a function of respective perceptual differences (Anstis, Howard, & Rogers, 1978; Loomis & Nakayama, 1973; Robinson, 1998; Oyama, 1977). Lateral inhibition caused by close spatial relations also has counterparts in other perceptual dimensions, *e.g.* orientation and size, as well as spatial dimensions (Blakemore, Carpenter, & Georgeson, 1970; Oyama, 1977). Phenomena like apparent motion can also be found even in a non-spatial auditory region, the pitch of tone (Yuki, 1965; Oyama, Torii & Mochizuki, 2005) Under some optimal frequencies of alternation of two tones, the observers get an impression of a tone moving up and down continuously in pitch, as trill in music. All of these perceptual phenomena can be represented and even predicted by our model.

Further elaboration of the perceptual state-space model is needed for these purposes. In the above discussion of the perceptual state space, we have treated many perceptual dimensions in the same way, but specific characteristics of individual perceptual dimensions should be reflected in a more elaborated model. Functional relations between these perceptual dimensions and their corresponding physical dimensions should also be studied more precisely.

Authors' Note

Parts of this study have been presented by the first author at the 21st Conference of the Japanese Psychonomic Society, Chiba, November 2003, the 28th International Congress of Psychology, Beijing, August 2004, the 50th Conference of the Japanese Society of Theoretical Psychology, Tokyo, November 2004, and the 23rd Meeting of the International Society for Psychophysics, Tokyo, October 2007.

The authors are grateful to Michael Kubovy, Shiro Imai, Masako Jitsumori, and Viktor Sarris for their useful comments on earlier drafts of this article and to Takashi Onuki, Chiba University for providing additional experimental data.

Summary

Quantification of the proximity and similarity laws was accomplished on the basis of the experimental results of Oyama, Simizu & Tozawa (1999) on perceptual grouping and apparent motion. Matched separations between the homogenous stimulus elements were obtained for various combinations of dissimilarities in hues, brightness, size and shape. A multiple linear regression analysis was applied to the obtained separations to compare the effects of dissimilarities among the various perceptual dimensions studied. Separations and dissimilarities worked additively across the different perceptual dimensions. Similarities were also compared with proximity in this process. A *perceptual-state-space model* was proposed, which consists of dimensions representing hue, brightness, saturation, size, shape, as well as three spatial dimensions. Perceptual grouping, apparent motion and perceptual changes correspond to interactions or displacements between points in the perceptual state space.

Keywords: Gestalt laws, proximity, similarity, quantitative model, regression analysis

Zusammenfassung

Gestützt auf Daten zur anschaulichen Gruppierung und Scheinbewegung (Oyama, Simizu & Tozawa, 1999) wurde versucht, die Gestaltgesetze der Nähe und Ähnlichkeit zu quantifizieren. Abstandsbeurteilungen zwischen homogenen Reizeinheiten (resultierend aus verschiedenen Kombinationen von Unähnlichkeiten zwischen Farbton, Helligkeit, Größe und Form) wurden einer multiplen linearen Regressionsanalyse unterzogen und somit die zwischen den Wahrnehmungsdimensionen auftretenden Unähnlichkeits-Effekte verglichen. Über die Dimensionen hinweg zeigten sich additive Abstands-Unähnlichkeits-Beziehungen. Entsprechend wurden Ähnlichkeiten mit Nähe verglichen. Als Modell ergab sich so ein Farbton, Helligkeit, Sättigung, Größe, Form sowie die drei Raumdimensionen enthaltender *Wahrnehmungs-Zustands-Raum*. Anschauliche Gruppierung, Scheinbewegung und Wahrnehmungsänderungen entsprechen Interaktionen oder Verschiebungen zwischen Punkten im Wahrnehmungs-Zustands-Raum.

References

Anstis, S. M., Howard, I. P. & Rogers, B. J. (1978): A Craik-O'Brien-Cornsweet illusion for visual depth. *Vision Research 18*, 213-217

Ashby, W. R. (1956): *An Introduction to Cybernetics*. London: Chapman & Hall.

Blakemore, C., Carpenter, R. H. S. & Georgeson, M. A. (1970): Lateral inhibition between orientation detectors in the human visual system. *Nature 228,* 37-39.

Hochberg, J. E. (1957): Effects of the Gestalt revolution: The Cornell symposium on perception. *Psychological Review 64*, 73-84.

Hochberg, J. & Hardy, D. (1960): Brightness and proximity factors in grouping. *Perceptual and Motor Skills 10*, 22.

Hochberg, J. & Silverstone, A. (1956): A quantitative index of stimulus similarity: Proximity versus differences in brightness. *American Journal of Psychology 69*, 456-458.

Johansson, G. (1958): Rigidity, stability and motion in perceptual space. *Acta Psychologica 14*, 359-370.

Koffka, K. (1935): *Principles of Gestalt Psychology.* London: Kegan Paul.

Kubovy, M. (1994): The perceptual organization of dot lattices. *Psychonomic Bulletin & Review 1*, 182-190.

Kubovy, M., Holcombe. A. O. & Wagemans, J. (1998): On the lawfulness of grouping by proximity. *Cognitive Psychology 35*, 71-98.

Kubovy, M. & Wagemans, J. (1995): Grouping by proximity and multistability in dot lattices, *Psychological Sciences 6,* 225-234.

Kubovy, M. & van den Berg, M. (2008): The whole is equal to the sum of its parts: A probabilistic model of grouping by proximity and similarity in regular patterns, *Psychological Review* (in press, January 2008 issue)

Loomis, J. M. & Nakayama, K. (1973): A velocity analogue of brightness contrast. *Perception 2,* 425-428.

Metzger, W. (1953): *Gesetze des Sehens*. 2. Aufl. Frankfurt a. M.: W. Kramer

Oyama, T. (1961): Perceptual grouping as a function of proximity. *Perceptual and Motor Skills 13*, 205-306.

Oyama, T. (1977): Feature analysers, optical illusions, and figural aftereffects. *Perception 6,* 401-406.

Oyama, T. (1997): Guest editorial: Apparent motion as an example of perceptual stability. *Perception 26*, 547-551.

Oyama, T., Miyano, H. & Yamada, H.(2003): Multidimensional scaling of computer- generated abstract forms. In H. Yanai, A. Okada, K. Shigemasu, Y. Kano & J. J. Meulman (Eds.): *New Developments in Psychometrics*, 551-558. Tokyo: Springer.

Oyama, T., Simizu, M. & Tozawa, J. (1999): Effects of similarity on apparent motion and perceptual grouping. *Perception 28*, 739-748.

Oyama, T., Torii, S. & Mochizuki, T. (2005): Pioneering studies in the 1930s on perception: A historical background of experimental psychology in Japan. *Japanese Psychological Research 47*, 73-87.

Robinson, J. O. (1998) : *The Psychology of Visual Illusion.* 2nd ed. Mineola, New York: Dover.

Sarris, V. (1989): Max Wertheimer on seen motion: Theory and evidence. *Psychological Research 51,* 58-68.

Wertheimer, M. (1923): Untersuchungen zur Lehre von der Gestalt II. *Psychologische Forschung 4,* 301-350.

Yuki, K. (1965): Experimental psychology. In A. H. Livermore (Ed.): *Science in Japan,* 335-373. Washington, DC: AAAS.

Zusne, L. (1970): *Visual Perception of Forms.* New York: Academic Press.

Addresses of the Authors:

Tadasu Oyama,

16-18 Oyamacho, Shibuyaku,

Tokyo, 151-0065 Japan

Email: oyama@gssc.nihon-u.ac.jp.

Hisao Miyano
Research Division
National Center for University Entrance Examinations,
Tokyo, Japan
Email: miyano@rd.dnc.ac.jp