

Isomorphism in Gestalt Theory: Comparison of Wertheimer's and Köhler's Concepts

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DEFINITION AND OVERVIEW

The term isomorphism literally means equality or sameness (iso) of form (morphism). In mathematics an isomorphism between two systems requires a one-to-one correspondence between their elements (that is, each element of one system corresponds to one and only one element of the other system, and conversely), which also preserves structures. Referring to isomorphism as one of the most important and general mathematical concepts, R. Duncan LUCE and Patrick SUPPES (1968, p. 72) characterize it as "a one-to-one mapping of a system A onto a system B in which the operations and relations of A are preserved under the mapping and have the same structure as the operations and relations of system B." In Gestalt psychology, the one-to-one correspondence between elements is not required; similarity of structures is required. What does isomorphism mean in Gestalt theory? To answer this question, we attempted to survey some of what had appeared in the Gestalt psychological literature (mainly in English) about isomorphism and related concepts. We cite the views of the founders of Gestalt theory and of a sample of other psychologists.

We begin with a historical remark by Kurt KOFFKA (1935), recalling his conversations with Max WERTHEIMER in 1911, shortly after the completion of experimentation on apparent movement in which Wolfgang KÖHLER and KOFFKA were the chief subjects. We do not know precisely what WERTHEIMER said, but he might have mentioned his hypothesis that the apparent movement, which he called the phi phenomenon, resulted from "a kind of physiological short-circuit" in the brain (1912b). KOFFKA was impressed by "the relation between consciousness and the underlying physiological processes, or, in our new terminology, between the behavioural and the physiological field." He noted that the statement in these new terms was made possible only by WERTHEIMER's idea. After referring to WERTHEIMER as the one who "first pronounced" the theory and KÖHLER as its elaborator, KOFFKA mentioned the principle of isomorphism, "according to which characteristic aspects of the physiological processes are also characteristic aspects of the conscious processes."

We then cite KÖHLER's references to isomorphism in some of his writings (e.g., 1920, 1929, 1938) and note his acknowledgement of the ideas of the co-founders of Gestalt psychology. His studies of physical Gestalten culminated in the hypothesis of psychophysical isomorphism.

Turning to Max WERTHEIMER, we first describe his work on the phi phenomenon and its significance (A.S. LUCHINS, 1968). Then we discuss lectures that WERTHEIMER gave in a 1937-1938 seminar at the New School for Social Research. He related isomorphism to perception of feelings, emotions, and expressive movements. He also pointed to differences between his and KÖHLER's conceptions of isomorphism. Our sources were the first author's notes on WERTHEIMER's lectures

and our reconstruction of the seminars (1973; 1991-1993). [Of the authors we cite, ARNHEIM, ASCH, SCHEERER, and the present first author, attended WERTHEIMER's seminars at the New School for Social Research]

Next we turn to Martin SCHEERER (1954) who, in a section on Gestalt psychology in a chapter on cognitive theory, raised the question of what determines the organizational character of a percept. He pointed to the Gestaltists' postulate of a dynamic self-distribution of nervous excitations triggered off by the proximal stimuli; this "culminated in KÖHLER's theory of isomorphism." SCHEERER noted that for the Gestaltist the total field consists of the geographic environment, which includes the psycho-physical organism; he also characterized the phenomenal field and the behavioural environment. Additionally, he pointed to some deficiencies or gaps in Gestalt psychological research, for example, the focus on the "palpably present behavioural environment" to the neglect of the environment which one imagines or thinks about. KOFFKA (1935) also had agreed that there were gaps in the research. Since 1935, there have been attempts to close the gaps, for example, by research and exposition on Gestalt principles applied to emotions, imagery, music, art, language, and thinking.

An example is Rudolf ARNHEIM's work on Gestalt theory applied to perception and art (1969). Another example is George HUMPHREY's writing in *Thinking* (1951) about psychoneural processes and isomorphism in Gestalt theory.

We then refer to two survey articles. In his encyclopedia article on Gestalt theory, Solomon ASCH (1968) discussed perceptual organization, as well as physical and physiological Gestalten. He also referred to WERTHEIMER's apparent movement study but not to the physiological short-circuit hypothesis; the only reference to isomorphism was to KÖHLER's psychophysical isomorphism.

Then we turn to the historian of psychology, Edwin G. BORING (1942, 1950), to consider what he wrote about the phi phenomenon, and about isomorphism and its relation to projection. BORING also described some criticisms of the isomorphism concept in Gestalt psychology and suggested that the future might show the validity of the criticisms, or put otherwise, the worth of the concept. We suggest that the future has arrived and that it is time to discuss the concept of isomorphism in Gestalt psychology.

A section entitled "Isomorphism, Phenomenology, and Beyond Phenomenology" refers to Giovanni VICARIO's description of his mentor, Gaetano KANIZSA, as a Gestaltist and experimental phenomenologist. We suggest that WERTHEIMER, who might have been influenced by phenomenology, was more oriented than KÖHLER to experimental phenomenology and less interested in physiological hypotheses. Such differences might help account for differences in their conceptions of isomorphism.

KOFFKA: PHYSIOLOGICAL BASIS OF ISOMORPHISM

In a section entitled "Relation Between Behavioural and Physiological Field Crucial," Koffka (1935, pp. 53-54) wrote about a conversation that

remains in my memory as one of the crucial moments of my life. It happened at Frankfurt on the Main early in 1911. WERTHEIMER had just completed his experiments on the perception of motion [phi phenomenon] in which KÖHLER and I had served as the chief observers. Now he proposed to tell me the purpose of his experiments ... [O]n that afternoon he said something which impressed me more than anything else, and that was his idea about the function of a physiological theory in psychology, the relation between consciousness and the underlying physiological

processes, or in our new terminology, between the behavioural and the physiological field. To state it in these new terms, however, is not quite fair, because this very statement was only made possible by WERTHEIMER's idea; before, nobody thought of a physiological or, for that matter, of a behavioural field.

KOFFKA criticized the theory of "merely molecular physiological processes." He maintained that, on the molar level, behaviour is not fundamentally different from the underlying physiological processes:

The assumption of merely molecular physiological processes is erected on much too slender an empirical basis; it results either in a molecular interpretation of behaviour, and consciousness, which is contradicted by the facts, or it severs completely the two series of processes, physiological and behavioural or conscious. (p. 56)

WERTHEIMER's Solution. Isomorphism. And now the reader can understand WERTHEIMER's contribution; now he will see why his physiological hypothesis impressed me more than anything else. In two words, what he said amounted to this: let us think of the physiological processes not as molecular, but as molar phenomena. If we do that, all the difficulties of the old theory disappear. For if they are molar, their molar properties will be the same as those of the conscious processes which they are supposed to underlie. And if that is so, our two realms, instead of being separated by an impossible gulf, are brought as closely together as possible with consequence that we can use our observations of the behavioural environment and of behaviour as data for the concrete elaboration of physiological hypotheses. (Ibid.)

On a subsequent page (p. 62) KOFFKA wrote:

if B stands for the behavioural world, G for the geographical, and P for the physiological processes, BP(G shows the relationship [If] B and P are essentially alike, then it only depends upon the G-P relation when and how we can gain about G from P. And if it is so, then surely observation of B reveals to us properties of P. This theory, first pronounced by WERTHEIMER, was carefully elaborated by KÖHLER. In his book on the "Physische Gestalten" (1920) he has gone deeply into physics and physiology to prove the compatibility of the theory with physical and physiological facts; in his "Gestalt Psychology" [1929] he has formulated this theory of isomorphism in a number of special axioms [and] the general principle in these words: "Any actual consciousness is in every case not only blindly coupled to its corresponding psychophysical processes, but is akin to it in essential structural properties" (p. 193). Thus, isomorphism, a term implying equality of form, makes the bold assumption that the "motion of the atoms and molecules of the brain" are not "fundamentally different from thoughts and feelings."

Later in the same text (p. 109), KOFFKA wrote:

"For we can at least select psychological organizations which occur under simple conditions and can then predict that they may possess regularity, symmetry, simplicity. This condition is based on the principle of isomorphism, according to which characteristic aspects of the physiological processes are also characteristic aspects of the corresponding conscious processes."

KÖHLER: PSYCHOPHYSICAL ISOMORPHISM

KÖHLER acknowledged the contributions of WERTHEIMER and KOFFKA. Referring to the close approach between general biology and psychology in the theory of nervous functions, particularly in the doctrine of the physical basis of consciousness, he wrote in his book on physical Gestalten (1920; abridged translation in ELLIS, 1938):

Here we have an immediate correspondence between mental and physical processes and the demand seems inescapable that at this point organic functions be thought of as participating in and exhibiting essentially Gestalt characteristics. The import and extraordinary significance of this was first recognized by WERTHEIMER who thereby attached to Gestalten a degree of reality far beyond any they had previously possessed. This implies, as KOFFKA emphasized, that central physiological processes cannot be regarded as sums of individual excitations, but as configured whole-processes. (1920/1938, p. 6)

The work of WERTHEIMER and KOFFKA has proceeded... in conformity with our earlier remarks about physical systems... It is the aim of this essay to support the WERTHEIMER hypothesis on physical grounds. (p. 20)

Discussing the behaviour of physical systems in their progress towards stationary states, KÖHLER concluded:

The law exemplified in cases of this sort may be called the tendency towards simple Gestalten, or the law of Prägnanz... This designation comes from WERTHEIMER, not as a description of inorganic physical behaviour, but of phenomenal and therefore also of physiological process-structures. Nevertheless it is possible to apply the terms to physical phenomena also, for the general tendency and line of development observed by WERTHEIMER in psychology and designated by him as the law of Prägnanz is obviously the same as we have here been discussing. (p. 54)

It is interesting that the term isomorphism did not occur in the index of KÖHLER's book, *Gestalt Psychology* (1929). Yet it occurred in a few places in the text, for example:

There is no reason at all why the construction of physiological processes directly underlying experience should be impossible, if experience allows us the construction of a physical world outside, which is related to it much less intimately... I should have ever so much difficulty in trying to relate definite experience to definite processes so long as I failed to assume one specific relationship between the two orders, viz., that of congruence or isomorphism in their systematic properties. (1929, p. 61)

KÖHLER added that the principle was sometimes formulated more explicitly in a number of "psychophysical axioms" (referring in a footnote to George E. MÜLLER, 1897, p. 189). But instead he gave examples to illustrate the principle.

The term isomorphism occurred frequently in another book by KÖHLER, *The Place of Value in a World of Facts* (1938):

... the most essential traits of experimental or perceptual contexts are the same as those of their physical counterparts. With respect to these traits the perceptual and the physical structures are isomorphic. If they were not, we could have no physics. (p. 162)

KÖHLER described many examples and concluded:

in all these cases it is really structure in which the world of percepts and the physical world have so much in common. Resemblance as to the demarcation of definite objects, and therefore to their number, means in fact similarity in the gross structure of the two worlds. And then inside such particular objects there is again structural resemblance between the perceptual and the physical world. (p. 166)

Physics, it was stated, proceeds on the assumption that certain structural traits of percepts agree with the structure of corresponding physical situations. It is, however, only macroscopic structures which can be common characteristics of the perceptual and the physical world. And this statement has sense only if the notion of macroscopic objects is found to refer to definite physical entities. We have, I believe, been able to show that it does. It is therefore a meaningful thesis that perceptual and physical contexts are isomorphic in essential macroscopic respects, and that to this extent there is resemblance between the phenomenal and the physical world, (p. 184)

In *The Place of Value in a World of Facts* (1938), KÖHLER has a chapter (Chapter VI) entitled, "On Isomorphism," from which we cite:

Concerning the emotional sphere, he wrote: "I propose to consider the nature of cortical processes although many philosophers dislike to hear much about the brain when philosophical problems are being discussed" (p. 185). "The cortical correlates of mental life or, as we may also call them, the psychophysical processes, are more interesting for our purposes than any other biological facts" (p. 194). "[It is not] a plausible assumption that cortical processes consist of independent events in individual cells. In the following paragraphs psychophysical correlates will, therefore, be considered from a macroscopic point of view" (p. 212). "Practically any part of human experience might be taken as an example of the fact that molecular events in the brain do not as such show much resemblance with phenomena" (p. 215).

Continuity is a structural trait of the visual field. It is also a structural fact that in this field circumscribed particular percepts are segregated as patches, figures, and things. In both characteristics, we have found, the macroscopic aspect of cortical processes resembles visual experience. To this extent, therefore, vision and its cortical correlate are isomorphic. In the last chapter the same term has been used. There, however, it applied to the relation between visual organization on the one hand and the macroscopic structure of situations in physical space on the other. The fact which mediates between the physical and the perceptual structure is now found to be cortical organization, which, as a rule, resembles both... Where perceptual organization does not agree with facts in physical space, cortical organization seems to agree with perception rather than with physics. (1938, pp. 217-218)

... Our present discussion is mainly concerned with the question of isomorphism between the visual field and its psychophysical correlate... Not for a moment should we forget, however, that isomorphism, thus considered, is a relation between visual experience and dynamic realities. (1938, pp. 218-219)

WERTHEIMER: THE PHI PHENOMENON AND ITS SIGNIFICANCE

The following comes from LUCHINS (1968):

WERTHEIMER sought examples from the field of perception, an area of psychology with a high reputation for exactness. He had little success until 1910, when he went on a trip, and while on the train, he thought of an optical phenomenon that seemed suitable. At Frankfurt he got off the train and bought a toy stroboscope. In a hotel room he set up the experiment by substituting strips of paper on which he had drawn series of lines for the pictures in the toy. The results were as he expected: by varying the time interval between the exposure of the lines, he found that he could see one line after another, two lines standing side by side, or a line moving from one position to another. This "movement" came to be known as the phi phenomenon.

WERTHEIMER asked SCHUMANN, his former teacher at Berlin and now at the Frankfurt Psychological Institute, if he could provide someone to act as an experimental subject.

SCHUMANN's laboratory assistant, Wolfgang KÖHLER came. For the next experimental session, KÖHLER brought his friend Kurt KOFFKA, who also served as a subject. KÖHLER persuaded SCHUMANN to visit WERTHEIMER and to invite him to conduct his experiment at the Frankfurt Institute. A simple apparatus to demonstrate the phi phenomenon was constructed, and the now classical experiment was conducted (WERTHEIMER, 1912b).

WERTHEIMER explained the significance of the experiment as follows: *"What do we see when we see the movements of a hand or a light? Is it appropriate to say that we have a sensation in different places on the retina from which movement is inferred? Is it appropriate to cut the phenomenon of movement in this way into a number of static sensations?"* (1937). Although there had been psychologists and philosophers before him who believed that movement was not an inference from static sensations on the retina but was a sensation sui generis, they had not demonstrated this in a scientific manner. WERTHEIMER now presented the thesis in a way which made experimental decisions possible.

It was not merely WERTHEIMER's experiment but his formulation of the underlying problem and the way to proceed to solve this problem that launched Gestalt psychology. Through experimental variations, he tested, one by one, various possible explanations of the phi phenomenon and found them wanting. According to WERTHEIMER, the essential features of the phi phenomenon are the following: it is a counter example to the assumption that piecemeal and summative approaches to psychological phenomena are universally adequate; it belongs to a category of genuine dynamic experience which must be understood in terms of dynamics rather than reduced to static events; finally, it is an example of a structure that is not an arbitrary arrangement of events but has inner connectedness (1937).

WERTHEIMER felt that there was a need for a model of such dynamic experiences, and he hypothesized a possible physiological process: *"The motion is due to a field of activity among cells... not excitation in isolated cells but field effects"* (1937). This model applied concepts of field-theoretical physics to a neurological event. (LUCHINS, 1968, pp. 523-525)

WERTHEIMER: ISOMORPHISM

This section is based mainly on LUCHINS and LUCHINS (1973, Vol. II, pp. 157-171), which reconstructed several sessions of WERTHEIMER's 1937-1938 seminar at the New School for Social Research that were devoted to "a new theory of perception of feelings." First WERTHEIMER described a lecture-demonstration in which the instructor was standing behind a table in the front of the room. In the back of the room were two boxes of the same size, shape, and color. The instructor asked a student to go to the back of the room and bring to his table, one at a time, each of the two boxes. The box contents were visible to the student who carried them, but not to the other students in the class. (One box held a sensitive, delicate apparatus, but the other held old

newspapers.) The class members were asked to describe verbally what they saw and also to draw graphs of the student's movements with each box. For example, when the student carried the box with the instrument, he and his movements were described as cautious and careful; when the student carried the newspapers, he and his movements were described as casual. The graphs that students drew showed Gestalt qualities of the behaviour.

WERTHEIMER described another demonstration: there were two rods with a wire between them on the instructor's table in front of the room. The instructor called on a student to touch the wire. After he did this, the experimenter made a motion as if to open a switch; the class members did not see the switch nor the words printed below it: DANGER HIGH VOLTAGE. (There was no danger to the student.) Again the student volunteer, who could see the words, was asked to touch the wire. The class was asked to describe what had happened, to characterize the student's behaviour each time, and to draw graphs of the actions. Descriptions of the volunteer's behaviour and the graphs showed hesitation and discontinuity after the instructor made a motion to open the switch. During a discussion WERTHEIMER made remarks such as these: If you say that someone is energetic or furious, are you referring to his behaviour, or to his feelings, or to both? In the old theory, feelings were considered separate from movements and other physical behaviour; they belonged to different, heterogeneous realms. The new theory recognizes that the Gestalt qualities of behaviour and feelings may be the same. He said, as KOFFKA (1935) had stated it: The same stimulus array that gives rise to seeing a face may contain the sadness that one sees in it. Isomorphism is a thesis that the Gestalt quality of psychological events is similar to the quality of the physical world. The old view held that if certain psychological feelings and certain physiological movements seem to be related, it is because they have been associated together in the past. WERTHEIMER wrote on the blackboard:

Old View	New View: Isomorphism
Psychology and physiology are similar because of association due to past experience.	Psychology and physiology are similar because of similar Gestalt qualities.

WERTHEIMER improvised on the piano and asked the class to match what he played with colors as well as with words that he had written on the blackboard, e.g., sad, happy, and aggressive. There was considerable agreement among the class members.

In response to a student's question about KÖHLER's concept of isomorphism, WERTHEIMER said that their formulations differ. He wrote on the blackboard:

- A) How stimulation is and how it is in the brain (K)
- B) How behaviour is and how it appears (W)

After a pause, WERTHEIMER said in reference to B) that isomorphism does not always hold. Some expressions of feeling are added to the behaviour, are external to it. Some expressions are due to cultural factors; in certain societies one has to hide feelings. Not all psychological states of a person are expressed in the responses and not all behaviour expresses the psychological states. He then raised the question: What are the criteria of real or true behaviour and of pretense or falsity? He suggested some methods of studying reactions to actual expressions of emotions and re-enactments of them.

In another comparison of the two conceptions of isomorphism (cf. SCHEERER, 1954), WERTHEIMER noted that KÖHLER assumes that phenomenal patterns correspond to patterns of the brain field's physiological processes. WERTHEIMER is concerned with the relationship between organization of the phenomenal field and that of the geographical field.

After class several seminar members continued the discussion of isomorphism. One student said

that he was surprised to hear that WERTHEIMER's thesis of isomorphism is not like KÖHLER's. From reading KOFFKA (1935) he had the impression that WERTHEIMER originated the thesis of neural physiological isomorphism. But it seems that WERTHEIMER actually focuses on the relation between the geographical and the phenomenal world, while KÖHLER focuses on the relation between the phenomenal and the psychophysical field. The discussants decided that neither KÖHLER nor WERTHEIMER would deny that it sometimes happens that the phenomenal, the geographical, and the brain field may be isomorphic.

Someone said that WERTHEIMER seems more concerned than KÖHLER with truth, with veridical perception. WERTHEIMER is aware of the lack of isomorphism in some cases and suggests that we find out the conditions that produce non-correspondence between the percept and the aspect of the real world that it maps. KÖHLER's thesis does not lead to such research.

Martin SCHEERER, a seminar member who had joined the discussion, said that WERTHEIMER's thesis of isomorphism does not require a physiological theory. WERTHEIMER is pointing out that there are resemblances between perception and the external world. A philosophy major argued that WERTHEIMER would have to be a Naïve Realist to believe that what we see actually exists; what we see is often not really so!

It was noted that psychology until recently was a branch of philosophy, and it has inherited the mind-body problem. A refugee scholar said that the principle of isomorphism is the Gestalt psychologists solution to the mind-body problem as it was formulated by DESCARTES. KÖHLER's thesis has replaced metaphysical dualism with an epistemological dualism. He went on to say that in many respects WERTHEIMER's thesis reflects SPINOZA's identity theory and PYTHAGORAS idea that Form is immanent in the material thing. ASL (A.S. LUCHINS) commented that WERTHEIMER's thesis of isomorphism is neutral to the question of the nature of the mind, the body, and the relation between them. His thesis calls for study of the relationship between the evidence and what one sees based on the evidence. The principles of organization provide an example of how WERTHEIMER would go about studying the relationship between the organization of the geographical and phenomenal fields. Someone recalled that WERTHEIMER said that the laws of organization also hold for structuring of the brain field. SCHEERER remarked that Gestalt isomorphism frees psychologists from the idea of mysterious conscious events in the brain. The philosophy major argued that WERTHEIMER himself used physiological speculation to concretize his thesis about a Gestalt as a dynamic whole in his short-circuit theory for phi movement. ASL pointed out that the neurologizing was a kind of footnote but WERTHEIMER himself claimed it was not essential to understanding of the phi phenomenon.

Someone remarked that WERTHEIMER's thesis takes us back to the pre-philosophical naïve view that there is a correlation between the structure of objects and the structure of psychological experience. ASL commented that WERTHEIMER does not say that there is always a correlation between the geographical and phenomenal worlds but that there are cases where there is such a correspondence; there are also cases where there is little or no correspondence. WERTHEIMER suggests that we study the factors involved in each case; what factors blind one and what factors open one's eyes to the structure of what is before him? Moreover, what can be done to bring about recovery for those who are blind to the structure? WERTHEIMER's thesis calls for empirical research. It involves looking for new ways to deal with the question of why things look the way they do.

The refugee scholar said that eventually WERTHEIMER's approach will have to reconcile itself to KÖHLER's because psychology is becoming a biological science. WERTHEIMER himself realized the value of a neurological model when he discussed the phi phenomenon. KÖHLER is merely following in the master's footsteps with the thesis of psychophysiological isomorphism. SCHEERER insisted that it is possible to deal with the percept and the geographical stimuli in WERTHEIMER's manner without a neurological model. ASL said that it may even be an advantage to do this; it will

spare Gestalt psychologists the criticism of both the traditional physiologists and the Behaviourists. The former object to Gestalt psychology, saying that the brain is a network of connections and not a field of interacting dynamic forces; the latter object to neurologizing of behaviour because they regard the brain as a mystery box. SCHEERER cautioned that we not overlook the differences between their views and WERTHEIMER's views on the nature of man. Someone added that they also differ in their concern with values and requiredness. KÖHLER also insists that we cannot avoid the question of value and agrees with WERTHEIMER that there are requirements of a system.

In other lectures WERTHEIMER contrasted KÖHLER's focus on brain dynamics and neurophysiology with his own interest in a mathematical model of isomorphism. WERTHEIMER did not accept the widespread belief that Gestalt psychology implies a neurophysiological model of brain dynamics. As we pointed out elsewhere (1991-1993), aside from the phi phenomenon, WERTHEIMER did not present such a model:

Unlike his paper (1912b) on the phi phenomenon, WERTHEIMER did not give a physiological model or any model of brain dynamics in his [other] publications [e.g., 1912a, 1925, or in his (posthumous) 1945 book on productive thinking] in discussing his concepts of centering, recentering, or restructuring. The notion that they are due to brain dynamics may have been reinforced by KÖHLER's discussion of insight [and] by HUMPHREY's (1951) and KOFFKA's (1935) references to WERTHEIMER's work in terms of brain dynamics. The fact is that WERTHEIMER, in his 1920 and 1945 works, and in his lectures on productive thinking [at the New School for Social Research], stressed the need for a new logic and new mathematics that would deal with Gestalten and Gestalt processes. He specifically pointed out in his lectures the differences between WERTHEIMER's and KÖHLER's concepts of isomorphism...and he contrasted his search for a mathematical model with KÖHLER's neurological model of isomorphism. (LUCINS & LUCINS, 1991-1993, Vol. II, p. 856)

SCHEERER: FIELDS AND ISOMORPHISM

In a chapter on cognitive theory, Martin SCHEERER (1954) wrote about the Gestaltists views on perception:

...the question [is] what determines the organizational character of a percept. It has been dealt with by the Gestaltists in two ways: first, they postulate a dynamic self-distribution of nervous excitations which is triggered off by the proximal stimuli. This culminated in KÖHLER's theory of isomorphism, which assumes a formal correspondence between brain-field patterns and phenomenal patterns; in the last analysis the latter are believed to derive from the former...Second, the Gestaltists examine the proximal stimulus conditions themselves as to their possible relevance for perceptual patterning. In so doing, KÖHLER concludes: "Although the local stimuli are mutually independent, they exhibit formal relations such as those of proximity and similarity" (1947, p. 1657). It is clearly implied that the WERTHEIMER factors give rise to perceptual unit formation. The organizational response is induced by the relations among the independent local stimuli in terms of similarity, proximity, common fate, and good continuation....The cognitive significance of these perceptual principles may be summed up in the proposition that phenomenal organization is a cohesive structured field and that the units in this field represent distal objects of the geographic environment. [The italicized sentence may be regarded as reflecting WERTHEIMER's concept of isomorphism.] In repudiating the traditional constancy hypothesis of a strict correspondence between local stimuli and sensation the theory has to postulate that something of the physical object is preserved which makes, on the whole, for its adequate representation....In the words of KOFFKA, the behavioural object, the perceptual representation of an object, "is a dynamic map of the distant stimulus when and inasmuch as the proximal stimulus distribution possesses such geometrical characteristics as will produce a psychophysical organization similar to the one of the distant stimulus object" (1935, p. 659). (SCHEERER, 1954, pp. 98-99)

The chapter also discussed the relationships among the geographic, the phenomenal, and the behavioural environments:

For the Gestaltists the total field, in the widest sense, consists of the geographic environment and the psycho-physical organism within it. Only a portion of this field is psychologically represented; this is the phenomenal (or psychological) field of direct experience or awareness. It includes distal representation and the phenomenal self. The behavioural environment, or life space, is more inclusive than the phenomenal field, and extends beyond momentary awareness. In turn, the behavioural environment corresponds only to that segment of the wide geographic environment that affects behaviour. (Ibid., p.100)

SCHEERER also pointed to some gaps in Gestalt psychological research:

The Gestaltists have not attempted to deal with the acquisition and function of symbols, concepts, ideational contents, and language, nor with the manner in which these are reflected in the phenomenal field and in motivated behaviour. KOFFKA acknowledges this gap. (Ibid., p. 113).

SCHEERER cited KOFFKA's comments (1935, p. 422) on the importance of not only the palpably presented behavioural environment but also the environment which we "merely" imagine or think of, an environment closely related to our language. Recognizing that "an ultimate explanation of the problems of thought and imagination will not be possible without a theory of language and other symbolic functions," KOFFKA admitted that "we shall exclude the study of language from our treatise," rather than risk a superficial treatment (1935, p. 422).

SCHEERER also pointed to an additional gap in Gestalt psychological research:

Another admitted gap is the social and value-oriented aspect of behaviour. Efforts have been made to implement Gestalt theory in this direction. With regard to value, KÖHLER (1938) has presented a novel approach of combining the phenomenological analysis of "requiredness" with the formal principles of isomorphism. ASCH (1952) has extended the Gestalt approach with the cognitive aspect of human interaction. However, in these later efforts, symbolic behaviour and its relation to motivation are secondary. (SCHEERER, 1954, p. 115)

The gaps that KOFFKA acknowledged in 1935 decreased in subsequent years. KÖHLER's 1938 book on value was acknowledged by SCHEERER. In the 1937-1938 seminars, WERTHEIMER dealt with the environment that one imagines and thinks of, as well as with emotions and with language and symbols. He provided lively demonstrations using colors and music and suggested experimental outgrowths. One of the most popular courses WERTHEIMER taught at the New School was on music and art. Some of his students undertook research in these areas. Among them was Rudolf ARNHEIM, who did a doctoral dissertation on expressive movement under WERTHEIMER, and who together with students did research on perception and art, resulting in his 1969 book to which we turn.

ARNHEIM: ART AND PERCEPTION

In *Art and Visual Perception: A Psychology of the Creative Eye*, Rudolf ARNHEIM (1969) acknowledged his indebtedness to Gestalt psychology and its founders:

The experiments I am citing and the principles of my psychological thinking derive largely from gestalt theory. This preference seems justifiable. Even psychologists who have certain quarrels with gestalt theory are willing to admit that the foundation of our present knowledge of visual perception has been laid in the laboratories of that school. But this is not all. From its beginning and throughout its development during the last half century, gestalt psychology has shown a kinship to art. The writings of Max WERTHEIMER, Wolfgang KÖHLER, Kurt KOFFKA are pervaded by it. Here and there in these writings the arts are explicitly mentioned, but what counts more is that the spirit underlying the reasoning of these men makes the artist feel at home. In fact, something like an artistic look at reality was needed to remind scientists that most phenomena of nature are not described adequately if they are analyzed piece by piece. The realization that a whole cannot be attained by adding up isolated parts was not new to the artist. (p. vii)

Concerned with dynamic patterns, and raising the question of what was meant by perceptual forces, ARNHEIM wrote:

Throughout this book it must be kept in mind that every visual pattern is dynamic. Just as a living organism cannot be described by its anatomy, so the essence of a visual experience cannot be expressed by ...static measurements [which]

define only the "stimulus," that is, the message sent to the eye by the physical world. But the life of a percept - its expression and meaning - derives entirely from the activity of the kind of forces that have been described. (p. 6) The reader may have noticed with apprehension the use of the term "forces." Are these forces merely figures of speech, or are they real? And if they are real, where do they exist?

They are assumed to be real in both realms of existence - that is, as psychological and as physical forces...In what sense can it be said that these forces exist, not only in experience, but also in the physical world? (Ibid.)

...Light rays, emanating from the sun or some other source, hit the object and are partly absorbed and partly reflected by it. Some of the reflected rays reach the lens of the eye and are projected on its sensitive background, the retina. Do the forces in question arise among the stimulations that light produces in the millions of small receptor organs situated in the retina? The possibility cannot be entirely excluded. But the receptor organs of the retina are essentially self-contained. In particular, the "cones," which are largely responsible for pattern vision, have little anatomical connection with each other, many of them having private pathways to the optic nerve.

In the brain center of vision itself, which is located in the back of the head, conditions seem to exist, however, that would allow for this very kind of process. According to gestalt psychologists, the cerebral area contains a field of electrochemical forces. These interact freely, unconstrained by the kind of compartmental division that is found among the retinal receptors. Stimulation at one point of the field is likely to spread to adjoining areas. As an example of a phenomenon that seems to presuppose such interaction, WERTHEIMER's experiments on illusory movement may be cited. If two light spots appear successively in a dark room for a split second, the observer often does not report two separate and independent experiences. Instead of seeing one light and then, at some distance, another, the observer sees only one light, which moves from one position to another. This illusory movement is so compelling that it cannot be distinguished from the actual displacement of one light dot. WERTHEIMER concluded that this effect was the result of "a kind of physiological short-circuit" in the brain center of vision, by which energy shifted from the place of the first stimulation to that of the second. In other words, he suggested that local brain stimulations acted upon each other dynamically. Subsequent research confirmed the validity of this hypothesis and provided more information about the exact nature and behaviour of cortical forces. Although all these findings were indirect, in that they inferred knowledge of physiological happenings from psychological observations, more recent investigations by KÖHLER have opened the way for the direct study of the brain processes themselves.

The forces that are experienced when looking at visual objects can be considered the psychological counterpart or equivalent of physiological forces active in the brain center of vision. Although these processes occur physiologically in the brain, they are experienced psychologically as though they were properties of the perceived objects themselves. (p 7)

Examples were given in the book to show that "simplicity requires a correspondence of structure between meaning and tangible pattern" (p. 51). ARNHEIM noted: "Such structural correspondence has been named isomorphism by gestalt psychologists" (Ibid.).

HUMPHREY: THINKING AND ISOMORPHISM

George HUMPHREY's book, *Thinking* (1951), has a chapter entitled "The Gestalt Theory of Thought" (Chapter VI, pp. 150-184). Under "General Characteristics of the Gestalt Theory," HUMPHREY wrote:

At the outset it should be pointed out that these wholes, which are to serve as psychological units, may be, and in fact characteristically are, extended in time. To borrow a phrase from the physicists they are four dimensional. As KÖHLER puts it, they are processes.. Thus the "phi-phenomenon" of WERTHEIMER, which may be called the experimental starting-point of the theory, is the experience of spatial motion over a certain period of time. For example, if two spots of light are thrown on a screen with suitable lengths of exposure and at suitable intervals of time and spatial distance, the observer sees not two stationary spots but one moving spot. This latter experience, according to the theory, cannot be analyzed into two discrete experiences corresponding to the patches of physical light on the screen. It is the experience of a single patch of light moving from this point in space to that. The experience corresponding to each stationary spot of light has been modified, and an entirely new kind of experience has been created, namely the phenomenal Gestalt of motion from one point to another. This "Gestalt of motion" is then "four dimensional." The same thing is true of musical notes. The experience corresponding to each such note is different according to the melody of which it forms part and to its place in the melody.... The whole gives the meaning to the "elements", and cannot be analyzed into them; for such analysis neglects the fact that when originally separate experiences are juxtaposed, with the result that a new Gestalt is formed, those original experiences lose their original character and acquire a fresh character from their membership in the new whole. Physically, a melody can of course be analyzed into so many discrete notes; psychologically it cannot. The melody-Gestalt [von EHRENFELS, 1890] is, again four

dimensional; it takes place in space and time....

From the elementary statement foregoing several points emerge. The Gestalt is new. Under the proper conditions, a new kind of experience is born out of the disjecta membra of relatively discrete experiences. The spots of light are experienced as two discrete spots, if the interval is properly chosen. WERTHEIMER states that an interval of about 0.03 sec. gave two simultaneous lines in perception, one of 0.2 sec. two successive lines, while one of about 0.06 sec gave motion [1925, p. 73]. The individual notes [of a melody] are experienced as separate notes if the intervals between them are sufficiently long. But under the proper conditions the melody experience arises, which is new. In the subsequent section, called "Isomorphism," HUMPHREY wrote about WERTHEIMER's physiological short-circuit hypothesis concerning apparent movement and about KÖHLER's concept of physiological isomorphism:

...it is claimed that between the experience and the physiological processes directly underlying it there is a specific relationship, of congruence or isomorphism, to use KÖHLER's term. Actual motion of a spot of light in the field of vision is presumably accompanied by some kind of neural displacement in the visual area of the brain. WERTHEIMER's hypothesis is that in the case of apparent movement, there is a similar shift of excitation from one centre to another in the brain, a physiological short-circuit [1925, p. 88]. Apparent motion and real motion thus have similar physiological correlates, namely actual neural displacement. This is generalized by KÖHLER into the statement that "experienced order in space is always structurally identical with a functional order in distribution of underlying brain processes," and similarly for time [1947, pp. 61-63, original 1929]. The same principle applies to the experience of totality, wholeness. An experienced whole, according to the theory, implies wholeness, totality in the underlying physiological process. If the melody is to be perceived as a unity there must be unity in the correlative physiological processes. And in general "units in experience go with functional units in the underlying physiological processes." Indeed, KÖHLER is prepared to extend the notion of isomorphism still further. Language, he points out, is the direct outcome of physiological processes in the organism. Hence, "It does not matter very much whether my words are taken as messages about experience or about these physiological facts" [Ibid., p. 64] (HUMPHREY, 1951, pp. 152-153)

Under the heading "General Statement of the Gestalt Theory of Thinking," he wrote:

...the Gestalt theory of thinking may be summarily stated as follows. There is first stimulation by the situation. This gives rise to a nexus of perceptual processes of a psycho-neural nature, which by dynamic interaction, with each other and with the mnemonic traces present, results in a re-ordering of the first perceptual processes, in the way which we call "seeing the problem" ... or "formulating the problem." At this stage, the psycho-neural process remains mainly at the perceptual level. Because of the dynamic interaction of the processes leading to it there has been a certain amount of transformation of the original perceptual material; but the stage is still provisional. Seeing the problem is only "a step toward solution." From the psycho-neural processes...springs a series of events which we call the thinking proper...Thus through the thought-processes the solution springs from the stresses inherent in the seen-problem, in a manner comparable to the way in which perception of "a spot in motion" springs from the stresses inherent in the psycho-neural ensemble of the phi-phenomenon experiment. The whole series of events, from seen-problem to solution, is then unitary. It is the series of events leading from one state to another of a self-regulating system under stress. This series is comparable to the total series of swings of a pendulum coming to rest, which is likewise of a unitary nature...It will be noticed that no attempt has been made to segregate neural events from those of experience. This omission is deliberate and is in accordance with the principle of isomorphism. (HUMPHREY, 1951, pp.

ASCH: ORGANIZATION AND ISOMORPHISM

Isomorphism in Gestalt psychology is sometimes regarded as strictly KÖHLER's psychophysical isomorphism. Thus, in Solomon E. ASCH's (1968) encyclopedia article on Gestalt theory, the only mention of isomorphism was to KÖHLER's psychophysical isomorphism.

KÖHLER proposed a fundamental change in the conception of cortical functioning-A region such as the optic sector may be considered an electrolyte; the processes within it occur according to physical laws of self-distribution rather than according to the microanatomy of neural networks. Local states of excitation are surrounded by fields that represent these states in their environment and interact with other local states similarly represented. On this basis KÖHLER put forward the hypothesis that there are physiological processes which are special instances of physicochemical gestalten and that these are the correlates of phenomenal gestalten.

Implicit in the preceding examination is the assumption of psychophysical isomorphism, or the proposition that brain processes include some structural features that are identical with those of organized experience. Isomorphism refers not to metrical but to topological correspondences; brain processes are assumed to preserve the functional relations of symmetry, closedness, and adjacency, not the exact sizes and angles of patterns projected on the retina. This

formulation diverges from the widely accepted view that phenomenal and physiological events are lawfully correlated but have no further likeness between them. The postulate of isomorphism is intended as a heuristic guide to investigation. In this manner KÖHLER sought a unified explanation for facts in neurophysiology and psychology among certain facts of physics. (ASCH, 1968, p. 161)

ASCH discussed the study of apparent movement but did not refer to the physiological short-circuit hypothesis, perhaps because he knew that WERTHEIMER regarded it as a footnote, rather than as essential in understanding the phi phenomenon. Under the heading of "Perceptual Organization," ASCH wrote:

... WERTHEIMER took the radical step of denying the reality of sensory elements as part of perceptual experience. His study of apparent movement ([1912-1920], 1925, pp. 1-105), which marks the formal beginning of gestalt theory, provides a specific illustration of this thesis WERTHEIMER pointed out that apparent movement is not a series of sensations but an effect of two stimulus events cooperating to produce a new, unitary outcome; perceived motion cannot be split up into successive stationary sensations.-From the assumption that experience consists of having one sensation followed by another, one cannot account for the experience of change inherent in motion, a conclusion that applies equally to the perception of real motion. (p. 159)

A further and more important step in this development was the gestalt account of grouping, or unit formation, in perception WERTHEIMER described certain fundamental principles of grouping, or unit formation, in perception, among them those of proximity, similarity, closure, common fate, and good continuation.-WERTHEIMER considered one principle, that of Prägnanz, fundamental and inclusive of the others. The principle of Prägnanz maintains that grouping tends toward maximal simplicity and balance, or toward the formation of "good form." (pp. 159-160)

Under the heading "Physical and physiological gestalten," ASCH wrote:

The concept of gestalt received a fundamental elaboration in the work of KÖHLER (1920, 1940). As a first step KÖHLER called attention to a striking similarity between certain aspects of field physics and facts of perceptual organization. He pointed to certain instances of functional wholes in physics that cannot be compounded from the action of their separate parts. There are macroscopic physical states that tend to develop toward an equilibrium and in the direction of maximal regularity. (p. 161)

Under "Nativism," ASCH wrote:

Gestalt theory holds that organization in accordance with general principles of physical dynamics is present from the start in psychological functioning. This position leaves wide scope for unlearned processes. At the same time, the widespread view that gestalt theory underestimates the effects of past experience is oversimplified. It is more important to note that the concept of organization determines the treatment of both unlearned and learned functions. Gestalt theory refers unlearned operations mainly to relationally determined physiochemical processes rather than to the action of specific anatomical structures. Similarly, it holds that the effects of past experience are also products of organization, or determined by structural requirements. (p. 169)

In the conclusion section, ASCH returned to isomorphism:

It is ... appropriate to stress that gestalt theory is not a completed system, that many of the issues it raised await resolution, and that it might be best described as a program of investigation or a region of problems. Thus, there is as yet little understanding of the physiological foundations that gestalt theory sought for psychology, and the postulate of isomorphism remains a heuristic principle. (p. 173)

BORING: ISOMORPHISM IN GESTALT PSYCHOLOGY

In his book, *A History of Experimental Psychology* (1950, original 1929), BORING wrote about the phi phenomenon and isomorphism in Chapter 13, entitled "Gestalt Psychology."

[In 1912] WERTHEIMER was describing seen movement under the conditions of discrete displacement of the stimulus, as it occurs in the stroboscope or in the cinema. Wundtian elementism would have required him to say that a sensation of given quality changes its location in time ... Such [apparent] movement is not sensation as the word has been used by WUNDT and KÜLPE. It could properly be called a phenomenon, as that word had been used by the phenomenologists, and thus WERTHEIMER called it phenomenal movement or simply the phi-phenomenon. Suppose you have a stimulus which is discretely displaced from position A to position B, and then back to A again, and

to B again ... If the time-interval between the exposures at the two positions is long, you see simply discrete displacement, no movement. If the time-interval is shortened, you begin to see some movement at A or at B or both. If it is still further shortened, you come to the optimal rate at which perfect movement back and forth is perceived between A and B and in which the phi-phenomenon marks the movement ... Such phi is a visual perception, localized in space, with given extension ... Phi is, moreover, an emergent. It pertains to a whole psychophysical situation and not to any of the separate factors that enter into it. In that it is as much a founded characteristic as is shape, melody, or any other Gestalt.

Because Gestalt psychology tends to deal with wholes it frequently finds itself concerned with fields and field theory. A field is a dynamic whole, a system in which an alteration of any part affects all the other parts Because perception seems often to follow laws of physical dynamics, KÖHLER has supposed that there are neural brain fields which underlie and account for the dynamics inherent in the phenomenon of perception. KOFFKA has supposed that you must understand human action in terms of a behavioural field which includes, not the stimuli and the physical environment, but the outer world and its objects as perceived and conceived by the actor. (1950, pp. 590-591) ... There is much more to be understood about Gestalt psychology than that it deals with wholes and phenomena. Usually it works in terms of field theory, as we have noted. The important Gestalt psychologists have accepted a special theory of relation between experienced phenomena and the underlying brain processes, the theory called isomorphism, and to that we shall return. (p. 593)

Returning to the isomorphism concept in Chapter 25, "Brain Function," BORING wrote:

In the twentieth century the Gestalt psychologists have argued for isomorphism. WERTHEIMER suggested this relation for seen movement in 1912, but KÖHLER has been its most effective supporter since 1920 Isomorphism is not projection but it implies it. The Gestalt theory is that a spatial pattern of perception is isomorphic with the spatial pattern of the underlying excitation in the brain. Isomorphic means corresponding topologically, but not topographically. Shapes are not preserved, but orders are. In-betweenness is preserved It seems pretty clear that WERTHEIMER and KÖHLER got this view, not from the results of research, but from the atmosphere of the times, perhaps from G.E. MÜLLER's axioms, which, like all axioms, ask for acceptance without proof. [Does history support BORING's conjecture? Was this conception of isomorphism not the result of research?] On the other hand, the belief in both visual and somesthetic cortical projection was growing and the two theories, projection and isomorphism, support each other. The stimulus-object and the peripheral excitation are isomorphic. The perception and the stimulus-object are isomorphic. If perception and the cortical excitation are isomorphic, then the cortical and peripheral excitation must also be isomorphic, since patterns isomorphic with the same pattern would be isomorphic with each other.

It is true that WERTHEIMER went out of his way to object to isomorphism between peripheral excitation and perception because he had in mind the many instances, like the perceptual constancies, where the correspondence is not exact topographically; but these arguments deal with gross approximations. There is no doubt that the reason that KÖHLER's contention seemed so plausible was due in part to the growth of the belief in projection. For the same reason some of KÖHLER's more recent experimental demonstrations of the isomorphic relation between perception and brain excitations are consistent with the theory of central projection or at least with central isomorphic reduplication if projection is not the physiological means which the organism employs. (Ibid., pp. 681-682)

The index to BORING's 1950 book listed projection, discussed on pages 680-682, only in relation to isomorphism. BORING's statements raise interesting questions about the relationship. Does isomorphism imply projection? Do projection and isomorphism support each other? Also of particular interest to us are BORING's remarks that WERTHEIMER went out of his way to object to isomorphism because he had in mind instances such as the perceptual "constancies," where the correspondence is not exact topographically; unfortunately, references were not cited.

For more detailed discussion of perception, BORING referred the reader to his 1942 book, *Sensations and Perceptions in the History of Experimental Psychology*. In Chapter 2, "Physiology of Sensations," the section entitled "Projection," included the following:

The receptor-fields of the sense-organs are projected upon the central nervous system in the sense that the afferent fibers lead to the central system. Indirectly by way of synaptic connections in nuclei, the tracts of all five senses establish in man connection with the cerebral cortex, although they also make other connections at subcortical reflex levels which do not involve the cortex. Thus neural anatomy has come to support Johannes MÜLLER's theory of specific nerve energies, which eventually became a projection theory of sensory quality. Sight is not hearing because the optic fibers are projected upon the occipital lobes and the auditory upon the temporal lobes. If you could cross-connect the optic and auditory nerves, you could, du BOIS-REYMOND imagined, see tones and hear colors. (p. 78). [fn 2]

BORING traced the history of projection in vision, noting that spatial differentiation has long been recognized as the basis of spatial perception:

The anatomy of the optic chiasma was known even to GALEN (ca. 175 A.D.) who explained the singleness of binocular vision by assuming that some of the optic fibers from each eye cross at the chiasma and join corresponding fibers from the other eye. The discovery of the horopter (the locus in space of points seen singly in binocular vision) by AGUILONIUS in 1613 certainly supported some such view, and NEWTON in 1717 assumed that half the fibers cross at the chiasma to join the corresponding fibers from the other eye either at the chiasma or at the brain. WOLLASTAN in 1824 observed hemianopia in himself; half the field of his vision disappeared when he was greatly fatigued, indicating that NEWTON's notion of visual projection was correct, that the fibers from the left halves of both retinas lead to the left half of the brain, and conversely. Thus, when [Johannes] MÜLLER came to the problem of vision in 1826, he had little choice about the matter. His nativism, furthermore, also led him to assume that spatial difference on the retina must mean spatial difference in the Sensorium. (pp. 79-80)

MÜLLER offered a formal theory of "specific sense energies" ... or "specific energies of nerves" ... In 1871 Julius BERNSTEIN offered an explicit projection theory involving sensory circles. Although there were many criticisms of his theory. BORING concluded: "Altogether the theory was convincing in 1871 and remains plausible today after seventy years" (p. 81).

It cannot be said, in fact, that we have advanced much beyond BERNSTEIN. It is no longer necessary to assume sensory circles at the periphery or irradiation at the center, for it is clear that all stimulation spreads in the peripheral organ, much or little according to its degree. In vision the spread is partly optical dispersion and partly retinal and neural spread: the brightest stars have the greatest magnitudes. On the skin, the dispersion comes about by way of pressure gradients or thermal gradients. In the inner ear, a loud tone affects more of the organ of Corti than does a weak tone. Thus central irradiation may be given up; but projection stands.

Projection is, of course, in this sense one-to-one. The receptor field and the cortical field are held to be isomorphic, that is to say, the spatial orders at the periphery are supposed to be reconstituted topologically in the brain - not the exact shapes, but the orders. (On the nature of isomorphism, see the next section.) ...

Some degree of isomorphic projection is also required by KÖHLER's theory of the isomorphic relation between perception and the patterns of excitation in the brain (see the next section). If the pattern of perception, being, in general, correct in spite of all the exceptions which Gestalt psychology has exhibited, resembles the pattern of stimulation, and if, as KÖHLER's theory asserts, it also resembles the brain pattern, why then the pattern in the brain must also resemble the pattern of stimulation. (pp. 82-83)

[The last sentence is reminiscent of WERTHEIMER's brief description of KÖHLER's isomorphism concept: How stimulation is and how it is in the brain (K). Note that BORING described projection as one-to-one. He went on to also describe isomorphism as one-to-one, whereas this point-to-point correspondence was not required in either WERTHEIMER's or KÖHLER's conception of isomorphism.]

The next section, called "Isomorphism," began as follows:

One system is said to be isomorphic with another in respect of their spatial relations if every point in the one corresponds to a point in the other and the topological relations or spatial orders of the points are the same in the two perception and stimulus are spatially isomorphic in as far as the perceived spatial orders correspond with the spatial orders in the stimulus. Projection of the stimulus field upon the cortex tends to be isomorphic If perception and brain field are both isomorphic with the stimulus field, they must be isomorphic with each other. It is to this solution of the mind-body problem that KÖHLER has applied the term isomorphism - meaning psychoneural isomorphism. The simplest test of such isomorphism is to see whether adjacencies and inbetweennesses are preserved from one system to the other.

This word has sometimes been extended to other sensory attributes than space. There would be temporal psychoneural isomorphism if the time-order of perceived events is the same as the time-order of the neural events underlying them. Intensive isomorphism would mean that sensory intensity always corresponds with degree of the total underlying excitation. Qualitative isomorphism, at which KÖHLER has hinted, implies that difference in sensory quality implies difference in excitatory quality, as if different kinds of ion-concentrations in the brain could explain the difference between yellow and blue or between sweet and sour - a rather improbable assumption in view of the uniformity of nervous actions.

Psychoneural isomorphism, however, is a special case of psychophysical parallelism and of the mind-body problem in general. It was believed to be axiomatic long before the anatomical and physiological knowledge of projection was sufficient to justify it. How did psychologists come to hold this view? Why did it seem axiomatic to them? (pp. 83-84)

BORING went on to conjecture about how KÖHLER came to the concept of psychoneural isomorphism:

When KÖHLER participated in the founding of Gestalt psychology (1920), he made over [George E.] MÜLLER's [1897] axioms in accordance with the new unanalytical dynamic conceptions. It was he, indeed, who applied the term isomorphism to this psychoneural relation, he and his colleagues who made the concept so important in Gestalt psychology that it is not always possible in their writings to distinguish between the phenomenal field and the correlated brain field. He was, nevertheless, explicit. The relationship is one of topological order, not of identity of size or shape (BORING, 1942, p. 90).

BORING continued by quoting KÖHLER's statements in his *Gestalt Psychology* (1929, pp. 64-66): all experienced order in space is a true representation of a corresponding order in the underlying dynamical context of physiological processes (p. 64)-experienced order in time is a true representation of the corresponding concrete order in the underlying dynamical context (p. 65). And the law for phenomenal organization is similar: to a context, experienced as "one thing" belonging together, there corresponds a dynamical unit or whole in the underlying physiological processes (p. 66). "In this respect again, the order of experience is a true representation of a corresponding functional order in the processes upon which it depends" (Ibid.).

BORING also discussed isomorphism in relation to WERTHEIMER's experiments on the phi phenomenon in a section called "Perceived Movement" (pp. 595-596). Using a tachistoscope, WERTHEIMER arranged for a single discrete displacement of a simple geometric object, such as a line segment.

The first member presented he designated a, the second b. When the time interval between a and b was relatively long (about 200 millisecc.), the subject perceived succession, first a, then b. When the interval was very short (less than 30 millisecc.), the perception was one of simultaneity, a and b together. In between successivity and simultaneity he got movement, the optimal interval for which was about 60 millisecc. [This gave rise to] the case of pure movement, which WERTHEIMER named phi-movement which connects the objects and has direction between them, but seems not in itself to be an object.

... For optimal movement one sees a single object moving, not an a turning into a b. In this contention WERTHEIMER was following out the tradition of MACH and EXNER, but he went further. He insisted on the validity of movement as an immediate experience without reference to basic constituents, on the "givenness" of and its irreducibility in terms of space and time. Out of such an intransigent phenomenology arose Gestalt psychology.

There is one other item of importance in WERTHEIMER's paper. He suggested that seen movement may be the consequence of a "physiological short-circuit" in the brain. Given exactly the right time-interval, the excitation at one point may be drawn over to become the excitation at the other, the process being - not a retinal process... - but a cortical process which is the physiological substrate of apparent movement. This form of psychophysical parallelism follows the axioms of MACH and G.E. MÜLLER, and anticipates the isomorphism of KÖHLER which has become so nearly an indispensable of Gestalt psychology.

WERTHEIMER's paper, supported presently by the enthusiasm of the growing school of Gestalt psychology, was a great success, for it was the starting point of well over a hundred papers on apparent movement during the next thirty years. At first there were but a few studies by Gestalt psychologists in Germany under the influence of KOFFKA and KÖHLER, but the Americans took up the topic in the 1920s with considerable zeal. (BORING, 1942, pp. 595-596) He noted: "*So well does WERTHEIMER's cortical short circuit fit the isomorphism of Gestalt psychology that KÖHLER elaborated and modified the theory in 1923, shortly after he had laid down his general principles for isomorphic brain fields in his *Physische Gestalten* of 1920*" (1942, p. 599).

In the notes BORING provided references, including those that contained criticisms, his own and others, of the concept of isomorphism in Gestalt psychology. [fn 3] He summarized some of the criticisms:

The criticism of psychoneural isomorphism has come from the physiologists and the positivistic psychologists. The physiologists said that the brain is, in general, a net-work of connections, not a field where dynamic forces, such as the Gestalt psychologists find in perception, can exist. The future will decide that point. Even the physiologists know little as yet about the action of the brain. The positivists said that a mere statement of correspondence between mind and body is not enough, that they wished to know how the one affects the other, and that operational analysis of the nature of the available evidence for isomorphism shows that the axioms should be rewritten so as to state relationships between neural events - between events in the brain and the other physiological events involved in the

description of experience. In this contention they are expressing a taste of scientific logic and again it is the future that will decide whether their preferences will be fruitful enough to persist. (BORING, 1942, p. 90)

It is now the future, almost six decades later. What does the reader think has been shown about whether the brain is a field with dynamic forces, or a network of connections, and whether isomorphism can be meaningful between neural and non-neural events and need not be limited to relationships between neural events?

ISOMORPHISM, PHENOMENOLOGY, AND BEYOND PHENOMENOLOGY

Some, but not all, of the sources we cited mentioned WERTHEIMER's physiological short-circuit hypothesis for apparent movement. Most of the citations referred only to KÖHLER's principle of isomorphism, usually described as psychophysical isomorphism.

WERTHEIMER would probably not be troubled that this principle was associated with KÖHLER. It seemed that initially both WERTHEIMER and KÖHLER were interested in the physiological or psychophysiological field. But subsequently WERTHEIMER became more concerned with the phenomenal and geographical fields. In his publications, aside from his classical study on apparent movement, he did not introduce a physiological model (but sought for a mathematical model of isomorphism). However, he did not entirely overlook physiology. In a 1937 lecture he talked about the "old view" which held that psychology and physiology (that is, psychological and physiological events) are similar because of associations due to past experience; the "new view" held that psychology and physiology are similar because of similar Gestalt qualities. He also said that he and KÖHLER had different formulations of isomorphism. He wrote: How stimulation is and how it is in the brain (K); how behaviour is and how it appears (W).

We think these words can be clarified by using the letters and schema that KOFFKA suggested. Let B represent the behavioural field, G the geographical (or physical) field, and P the physiological field. Consider the schema BP(G). His formulation of the principle of isomorphism as "characteristic aspects of the physiological processes are also characteristic aspects of the corresponding conscious processes" (1935, p. 109) focused on the relationship between B and P, which he called crucial (p. 53). It might be said that WERTHEIMER focused on the relationships between the behavioural and the geographical world, between B and G, whereas KÖHLER focused on the relationship between the behavioural and the physiological field, between B and P. Both apparently recognized that it sometimes happens that the behavioural field, the geographical field, and the physiological or brain field may be isomorphic, may have similar molar characteristics.

WERTHEIMER stated that isomorphism did not always hold and that the conditions under which it did or did not hold called for research. WERTHEIMER's thesis called for empirical research, for new ways to deal with the question of why things look the way they do. WERTHEIMER also broadened the concept of isomorphism beyond the perception of things to the perception of emotions, movements, language, and other symbols. KÖHLER considered such ideas in his 1938 book on values, which frequently referred to isomorphism.

It seems that there is more than one concept of isomorphism in Gestalt theory to judge by WERTHEIMER's distinction between his and KÖHLER's formulations. (We are reminded of the distinction drawn by GRELLING & OPPENHEIM (1991, 1988, original 1939) between two conceptions of Gestalt: Gestalt as configuration, shape or form, contrasted with Gestalt as "functional whole.>"). Neither KOFFKA, KÖHLER nor WERTHEIMER claimed to have introduced the concept of isomorphism in Gestalt theory. Nor is it easy to distinguish clearly between the contributions of one or the other of the founders. We are reminded of what Giovanni B. VICARIO (1994) wrote in a tribute to his mentor, Gaetano KANIZSA:

Curiously enough, Gestalt psychologists are a Gestalt in their own, since you will never be able to make a sure distinction between the ideas that are due to the one and the ideas that are due to the other: Everyone who is acquainted with the papers by WERTHEIMER, KÖHLER and KOFFKA knows very well that they constitute a unique book. In addition to, I noticed the same sort of unselfishness as to the ownership of research starting points that I can testify in the case of KANIZSA. (p. 127)

From his mentor, VICARIO wrote (p. 129), he learned the phenomenological attitude so well described by Wolfgang METZGER (1963, p. 12): *"to simply accept the facing thing as it is ... to let the thing speak for its own, without indulging in what we know, or we previously learned, or in what is obvious, in the knowledge of the subject, in logical demands, in linguistic prejudices ..."* and so on.

... I always appreciated his theoretical minimalism, cause or effect of his exclusive attention for the data of immediate experience ... I saw KANIZSA always attentive to single perceptual phenomena, and in some ways careless of their theoretical arrangement. He was a Gestaltist, I think, because he was an experimental phenomenologist, but during the years I saw him abandon the cumbersome physiological hypotheses of KÖHLER and KOFFKA, and retire to his own view of pure phenomena bounded by phenomenal laws. (p. 130)

METZGER dedicated his *Gesetze des Sehens* (1975; first edition, 1936) as follows: *"Dem Andenken Max WERTHEIMERS und den italienischen und japanischen Freuden, in denen sein Geist lebendig geblieben ist."* [In memory of Max WERTHEIMER and of Italian and Japanese friends, in whom his spirit is still alive.] Recalling that WERTHEIMER studied with Christian von EHRENFELS in Praha [Prague], with Carl STUMPF in Berlin, and with Friedrich SCHUMANN in Frankfurt, VICARIO surmised:

That should mean that he was a follower of [Franz] BRENTANO's phenomenology ... According to [Georges] THINÈS (1977) the neurophysiological hypotheses (isomorphism) with which WERTHEIMER and KÖHLER stuffed phenomenal evidence represent a backward step in the evolution of phenomenology ... Anyway, I think that, when speaking of the "spirit of WERTHEIMER", METZGER refers to the commandment that [Edmund] HUSSERL synthesized in the phrase: *Zurück zu den Sachen selbst!* [Back to the things themselves!] ... In short, phenomenology, and experimental, if possible. Exactly what KANIZSA did along his whole scientific life. (p. 130)

WERTHEIMER had what might be called a phenomenon-centered experimental approach to research: letting the phenomenon "speak for itself" and studying it under a variety of conditions [fn 4]. He issued an invitation for experimentation, not argumentation. [Years later KÖHLER issued a similar invitation in his Presidential address to the American Psychological Association (1959).] WERTHEIMER might have been influenced by HUSSERL's phenomenology. An essay on HUSSERL by Joseph LYONS (1968) stated:

The most direct and specific of HUSSERL's effects on psychology as may be expected, occurred in Europe Of the important group who were at the University of Berlin just before World War I, and from whose joint efforts came the school of gestalt psychology, Max WERTHEIMER and Karl DUNCKER were apparently deeply influenced by phenomenology. (p. 30)

Similarly, an essay by Robert B. MACLEOD (1968) on phenomenology, referred to the phenomenological approach to perceptual organization represented by the Berlin group of gestalt psychologists.

The reality of gestalt qualities had been recognized by Christian von EHRENFELS in his article, "Über Gestaltqualitäten" (1890) and by [Karl] STUMPF, but it was Max WERTHEIMER's experimental studies of apparent movement (1912) which set the stage for the gestalt movement. The older theories could not admit as psychologically valid an experience of movement when there is no physical movement in the stimuli; phenomenal movement had to be explained away as an illusion. WERTHEIMER, like [David] KATZ, simply accepted the phenomenal fact as valid, insisting that movement as such must have its direct neural correlate; hence the controversial principle of isomorphism While the gestalt theories which emerged, notably the physiological and the psychological field theories, go beyond phenomenology, the basic approach is in each case phenomenological. (p. 71)

While not denying the basic importance of phenomenology, KÖHLER maintained that it was necessary to go beyond phenomenology and consider the brain field. "Beyond Phenomenology" is

the title of Chapter IV in his 1938 book, *The Place of Value in a World of Facts*. The chapter began, "It is not our intention to restrict this investigation to questions of phenomenological description" (p. 102). Although he noted that "all questions of fundamental principle ... can only be solved on phenomenological grounds" (*ibid.*), he was concerned with transcending phenomena, with "transphenomenal reality." "Physical nature is generally believed to be of transphenomenal existence" (p. 104). "No matter what our epistemological convictions are, we must recognize, besides pure phenomenology, all the natural sciences, such as physics, chemistry, geology, biology" (p. 106). He added: "only such percepts serve the physicist's purpose as are trustworthy signs of transphenomenal reality" (p. 107). "Practically all research in natural science proceeds, I believe, on the tacit assumption that its subject-matter exists outside the phenomenal world of all observers" (p. 121). Discussing memory, he wrote:

There is only one part of nature which, according to present knowledge, could in this case be intimately in contact with phenomenal data. This part of nature is the circumscribed world of brain-events ... Our conclusion will therefore be that, in trying to remember something and knowing that we know it, our reference is from the point of science reference to a definite neurological, or better: neural entity. (p. 123)

Thus, although one might have been more influenced than the other by phenomenology, both KÖHLER and WERTHEIMER recognized the importance of the phenomenal field. It might be said that KÖHLER focused on its relation to the brain field. WERTHEIMER did not criticize this focus, but apparently was less interested in physiological speculation than in experimental investigations of the relationships between the behavioural and the geographical fields. Such differences in orientation, we conjecture, might help to explain their different conceptions of isomorphism.

SUMMARY

What does the word "isomorphism" mean in Gestalt psychology? To attempt to answer the question, a sample survey was undertaken of this concept and related ideas, mainly in the Gestalt psychological literature in English. Our interest was in what had been expressed about isomorphism, the phi phenomenon (whose study is considered to have suggested the concept and to have launched the experimental basis of Gestalt theory), and the relationships among various fields or environments in psychology, including the physiological or brain field. We presented excerpts from some of the writings by the founders of Gestalt psychology: KOFFKA, KÖHLER, and WERTHEIMER, as well as from some of the latter's lectures in seminars at the New School for Social Research, as revisited by the present authors. Also considered were reports by others who had attended WERTHEIMER's seminars: SCHEERER on fields and isomorphism; ARNHEIM on the psychology of perception and art; and a survey article by ASCH on Gestalt theory. We also presented excerpts from HUMPHREY on the Gestalt psychology of thinking. We described BORING's historical accounts of Gestalt psychology and the links that he drew between isomorphism and projection.

WERTHEIMER had referred to differences between his conception and KÖHLER's conception of isomorphism. The discussion section cited VICARIO's description of his mentor, KANIZSA, as an experimental phenomenologist who abandoned "the cumbersome physiological hypotheses of KÖHLER and KOFFKA." We suggested that WERTHEIMER had more of an experimental phenomenological approach, which fostered his concern with the relationship between the behavioural and the geographical fields, and less interest in physiological hypotheses than KÖHLER who was concerned with going "beyond phenomenology" to the physical world and ultimately to the brain field. Such differences might help account for differences in their conceptions of isomorphism.

ZUSAMMENFASSUNG

Was bedeutet das Wort "Isomorphie" in der Gestaltpsychologie? Um diese Frage zu beantworten, wird in dieser Untersuchung - vorwiegend in der englischsprachigen gestaltpsychologischen Literatur - der Verwendung dieses Konzepts und damit zusammenhängender Ideen nachgegangen. Das Interesse der Autoren ist dabei vor allem darauf gerichtet, was über Isomorphie, über das Phi-Phänomen (dessen Untersuchung als Ausgangspunkt für die Entwicklung des Isomorphie-Konzepts betrachtet wird und das für die experimentelle Fundierung der Gestalttheorie den Grundstein legte) und über die Beziehung zwischen verschiedenen Feldern oder Umgebungen, einschließlich des physiologischen oder Gehirnfeldes, in der Psychologie ausgesagt wird. Dabei werden Auszüge aus einigen Schriften der Begründer der Gestaltpsychologie, KOFFKA, KÖHLER und WERTHEIMER präsentiert sowie auch Auszüge aus WERTHEIMERS Vorlesungen in Seminaren an der New School for Social Research, die von den Autoren dieses Beitrags dokumentiert wurden. Auch Ausführungen anderer Besucher dieser Seminare werden dabei miteinbezogen: SCHEERERS Äußerungen über Felder und Isomorphie, ARNHEIMS Gedanken zur Psychologie der Wahrnehmung und der Kunst, ein Beitrag ASCHs über Gestalttheorie. Weiters werden Auszüge aus HUMPHREYS Arbeit über die Gestaltpsychologie des Denkens und BORINGS historischer Beitrag über die Gestaltpsychologie (und die Verbindungen, die er zwischen Isomorphie und Projektion herstellt) präsentiert.

WERTHEIMER selbst hat Unterschiede zwischen seiner und KÖHLERS Konzeption von Isomorphie angesprochen. Im Diskussionsteil dieses Beitrags wird VICARIO zitiert, der von seinem Mentor KANIZSA sagt, dieser hätte in seiner Entwicklung als experimenteller Phänomenologe "die Bürde der physiologischen Hypothesen von KÖHLER und KOFFKA" abgeworfen. Die Autoren kommen zu dem Schluß, daß für WERTHEIMER der experimentelle phänomenologische Ansatz im Vordergrund stand. Dieser veranlaßte ihn zur vornehmlichen Beschäftigung mit der Beziehung zwischen dem behaviouralen und dem geographischen Feld. Sein Interesse an physiologischen Hypothesen war daher geringer als bei KÖHLER, dessen Anliegen es war, über die Phänomenologie hinaus zu Aussagen über die physikalische Welt und letztlich zum Hirnfeld zu kommen. Die Beachtung dieser Unterschiede in ihrem Forschungsinteresse kann zu einem besseren Verständnis der Unterschiede in den Isomorphie-Konzeptionen von WERTHEIMER und KÖHLER beitragen.

[fn 2]: Thus, the composer Olivier MESSIAEN, speaking of the union of color and tone in his music, explained to an interviewer: "When I hear music, I see inwardly, in the mind's eye, colors which move with the music. This is not imagination, nor is it a psychic phenomenon. It is an inward reality." And Carol STEEN, a New York artist who, like most synesthetes, has had synesthetic experiences from an early age and who uses her perceptions in her work, says she distinguishes different types of headaches by their colors. "If it's a sinus headache, it's green," Ms. STEEN said. Synesthesia received a flurry of attention from artists and psychologists at the turn of the century. But until relatively recently, modern science largely ignored it. Those who experienced synesthesia rarely complained. And the private nature of the perceptions made investigation difficult - there was no objective way to tell what, if anything, unusual was taking place. In the past 10 years, however, the arrival of imaging techniques and other new technologies for studying the brain at work is revived interest in synesthesia, capturing the interest of a small core of researchers in a variety of countries and disciplines, PET scanners, electrophysiological recording, DNA analysis and other techniques are increasingly being used. In the current issue of *The Journal of Neuropsychiatry and Clinical Neurosciences*, for example, German Researchers from the University of Hanover Medical School report electrophysiological findings from a group of synesthetic subjects. An understanding of synesthesia as a perceptual anomaly, researchers hope, may eventually help elucidate normal perception, or even shed light on consciousness itself.

Meanwhile, much more remains unknown about the comingling of the senses than is known. Even basic facts about synesthesia - its prevalence, for example - are still less than certain.

[fn 3]: For the development of the psychophysical axioms into the isomorphic principle, BORING (1942, p. 96) offered a list of references that included the following: R.H. LOTZE, *Medicinische Psychologie*, 1852, pp. 206-232, especially pp. 230-232; H. GRASSMANN, *Zur Theorie der Farbenmischung*, *Ann. Phys. Chem.*, 165, 1853, pp. 69-84 (Eng. trans., *Phil. Mag*, 4 ser., 7, 1854, pp. 254-264); as well as writings of E. MACH (1865), E. HERING (1878); F.C. DONDERS (1881); G.E. MÜLLER (1896); M. WERTHEIMER (1912); W. KÖHLER (1920, 1929, 1938); and K. KOFFKA (1935).

Also mentioned by BORING (1942, p. 96) were his reports that gave criticisms as well as a positivistic restatement of isomorphism: Psychophysical systems and isomorphic relations, *Psychol. Rev.*, 43, 1936, pp. 565-587, especially pp. 579-586; A psychological function is the relation of successive differentiations of events in the organism, *Psychol. Rev.*, 44, 1937, pp. 445-461, especially pp. 454f; An operational restatement of G.E. MÜLLER's psychophysical axioms, *Psychol. Rev.*, 48, 1941, pp. 457-464.

[fn 4]: We adapted WERTHEIMER's approach in our phenomenon-centered variational approach. This orientation is reflected in our research on *Einstellung*, e.g., our 1959 book, *Rigidity of Behaviour: A Variational Approach to the Effect of Einstellung*. The preface notes: "Our research has been guided by a phenomenon-centered variational approach ... that involves centering on a specific phenomenon of behaviour and attempting to vary systematically the conditions under which it is studied" (p. x). The book ends with Chapter XXIV, "The need for a phenomenon-centered variational orientation." This orientation was also used in our other research, for example, "A variational approach to phenomena in social psychology" (1957).

References

- ARNHEIM, R. (1969). *Art and Visual Perception: A Psychology of the Creative Eye*. Berkeley-Los Angeles: University of California Press.
- ASCH, S.E. (1968). Gestalt theory. In D.L. SILLS (Ed.), A. JOHNSON (Hon. Ed), Vol. 6, pp. 158-175.
- BORING, E.G. (1942). *Sensation and Perception in the History of Experimental Psychology*. New York: D. Appleton-Century.
- BORING, E.G. (1950). *A History of Experimental Psychology*, 2nd ed. New York: Appleton-Century-Crofts.
- EHRENFELS, C. Von (1890). Über Gestaltqualitäten. *Vierteljahresschrift für wissenschaftliche Philosophie*, 14, pp. 249-292.
- ELLIS, W.D. (1938). *A Source Book of Gestalt Psychology*. London: Routledge & Kegan Paul.
- GRELLING, K. & OPPENHEIM, P. (1999, 1988, 1939). Logical analysis of "Gestalt" as "functional whole." Preprinted for members of the Fifth International Congress for the Unity of Science, Cambridge, MA (1939). (Scheduled to appear in *Erkenntnis*, 9, 1938/1939, pp. 70-76, but issue was not published.) In B. SMITH (Ed.) *Foundations of Gestalt Theory* (1988), Philosophia Resources Library, München-Wien. In *Gestalt Theory*, 21(1), pp. 49-54.
- HUMPHREY, G. (1951). *Thinking: An Introduction to its Experimental Psychology*. London: Methuen.
- KOFFKA, K. (1935). *Principles of Gestalt Psychology*. New York: Harcourt, Brace.
- KÖHLER, W. (1920). *Die physischen Gestalten in Ruhe und im stationären Zustand. Eine naturphilosophische Untersuchung*. Erlangen: Philosophische Akademie. Abridged translation in W.D. ELLIS (1938), pp. 17-54
- KÖHLER, W. (1923). *Zur Theorie der stroboskopischen Bewegung*. *Psychologische Forschung*, 3, pp. 397-406.
- KÖHLER, W. (1929). *Gestalt Psychology*. Rev. ed. 1947. New York: Liveright.
- KÖHLER, W. (1938). *The Place of Value in a World of Facts*. New York: Liveright.
- KÖHLER, W. (1940). *Dynamics in Psychology*. New York: Liveright.
- KÖHLER, W. (1959). Gestalt psychology today. *American Psychologist*, 14, pp. 727-734.
- LUCE, R.D. & SUPPES, P. (1968). Mathematics. In D.L.SILLS (Ed.), A. JOHNSON (Hon. Ed.), Vol. 10, pp. 65-76.
- LUCHINS, A.S. (1957). A variational approach to phenomena in social psychology. In M. SHERIF and M.W. WILSON (Eds.), *Emerging Problems in Social Psychology*, Chapter 5. Norman, OK: University Book Exchange.
- LUCHINS, A.S. (1968). Max WERTHEIMER. In D.L.SILLS (Ed.), A. JOHNSON (Hon.Ed.), Vol. 16, pp. 522-527.
- LUCHINS, A.S. & LUCHINS, E.H. (1959). *Rigidity of Behaviour: A Variational Approach to the Effect of Einstellung*. Eugene, OR: University of Oregon Books.
- LUCHINS, A.S. & LUCHINS, E.H. (1973). WERTHEIMER's Seminars Revisited: Problems in Perception, Vol. II. Psychology Department Reports. Albany, New York: State University of New York at Albany.
- LUCHINS, A.S. & LUCHINS, E.H. (1991-1993). *Max WERTHEIMER's Life and Background: Source Materials*, Vol. II. Preliminary ed. Troy, New York: Rensselaer Polytechnic Institute.
- LYONS, J. (1968). HUSSERL, Edmund. In D.L. SILLS (Ed.), A. JOHNSON (Hon. Ed.) Vol. 7, pp. 27-31.

- MACLEOD, R.B. (1968). Phenomenology. In D.L. SILLS (Ed.), A. JOHNSON (Hon. Ed.), Vol. 12, pp. 68-72.
- METZGER, W. (1963). Psychologie. Darmstadt: Steinkopff.
- METZGER, W. (1975). Gesetze des Sehens, 3rd ed. Frankfurt an Main: Kramer. (1st ed. 1936).
- MÜLLER, G.E. (1897). Zur Psychophysik der Gesichtsempfindungen. *Zeitschrift für Psychologie*, 14, pp. 1-76, pp. 161-196.
- SCHEERER, M. (1954). Cognitive Theory. In G. LINDZEY (Ed.), *Handbook of Social Psychology*, Vol. I, pp. 91-142.
- SILLS, D.L. (Ed.), JOHNSON, A. (Hon. Ed.) (1968). *The International Encyclopedia of the Social Sciences*. New York: Macmillan and The Free Press.
- THINÈS, G. (1977). *Phenomenology and the Science of Behaviour: An Historical and Epistemological Approach*. London: G. Allen & Unwin.
- VICARIO, G.B. (1994). Gaetano KANIZSA: The scientist and the man. *Japanese Psychological Research*, 36(3), pp. 126-137. Special Issue: Gestalt Perception I.
- WERTHEIMER, M. (1912a). Über das Denken der Naturvölker: I. Zahlen und Zahlgebilde. *Zeitschrift für Psychologie*, 60, pp. 321-378. Reprinted in *Drei Abhandlungen zur Gestalttheorie*, 1925. Abridged translation in W.D. ELLIS (1938), pp. 265-273.
- ERTHEIMER, M. (1912b). Experimentelle Studien über das Sehen von Bewegung. *Zeitschrift für Psychologie*, 61, pp. 161-265. Reprinted in *Drei Abhandlungen zur Gestalttheorie*, 1925.
- WERTHEIMER, M. (1920). Über Schlussprozesse im produktiven Denken. Berlin: De Gruyter, Pp. 22. Reprinted in *Drei Abhandlungen zur Gestalttheorie*, 1925. Abridged translation in W.D. ELLIS (1938), pp. 274-282.
- WERTHEIMER, M. (1925). *Drei Abhandlungen zur Gestalttheorie*. Erlangen: Philosophische Akademie. Contains reprints of *Über das Denken der Naturvölker*; *Experimentelle Studien über das Sehen von Bewegung*, and *Über Schlussprozesse im produktiven Denken*.
- WERTHEIMER, M. (1945). *Productive Thinking*. Enlarged ed. 1959, Michael WERTHEIMER (Ed.). New York: Harper.